Exercises in Algebraic Topology

Prof. Dr. Birgit Richter

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No 5 Due: Tuesday, 18th of May 2010

17 Let X be a CW complex and let X^n be the n-skeleton of X. Prove the universal property of the direct limit topology on X.

Show that X is homeomorphic to $\bigsqcup_{n\geqslant 0} X^n/\sim$ where the equivalence relation on $\bigsqcup_{n\geqslant 0} X^n$ identifies an $x \in X^n$ with $i_n^m(x) \in X^m$ where $i_n^m\colon X^n \to X^m$ is the inclusion of skeleta for $m \geqslant n$.

(4 points)

18 Write down explicit CW structures (cell decomposition and characteristic maps) for the torus, the Möbius strip and the Klein bottle.

(3 points)

19 Prove that for any subcomplex $A \subset X$ there is an open neighborhood U of A in X together with a strong deformation retraction to A. (This is not that easy. If you have problems, then at least try to understand a proof given in any decent book on algebraic topology.)

(4 points)

20 Let $f\colon \mathbb{S}^n \to \mathbb{S}^n$ be the restriction of an orthogonal linear map g of $\mathbb{R}^{n+1}$. Why is $\mathrm{grad}(f) = \det(g)$?

(1 point)