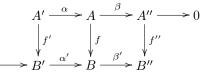
Exercises in Algebraic Topology

Prof. Dr. Birgit Richter Summer term 2010

Due: Tuesday, 27th of April 2010

5 Prove the snake lemma: If

No 2



 $0 \longrightarrow B' \xrightarrow{\alpha} B \xrightarrow{\beta} B''$ is a commutative diagram with exact rows, then there is an exact sequence

$$\ker(f') \to \ker(f) \to \ker(f'') \xrightarrow{\delta} \operatorname{coker}(f') \to \operatorname{coker}(f) \to \operatorname{coker}(f'')$$

Here, δ is a suitable connecting homomorphism; all other maps are induced ones.

6 The fundamental group of a connected, compact, closed and orientable surface F_g of genus $g \ge 1$ is

$$\pi_1(F_g) \cong \langle a_1, b_1, \dots, a_g, b_g | a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_g b_g a_g^{-1} b_g^{-1} \rangle.$$

Calculate $H_1(F_g)$ via the Hurewicz homomorphism.

7 Recall, that $i: A \hookrightarrow X$ is a strong deformation retract, if there a map $H: X \times [0, 1] \to X$ such that

- (1) H(x,0) = x for all $x \in X$,
- (2) $H(x,1) \in A$ for all $x \in X$,
- (3) H(a,t) = a for all $a \in A$ and $t \in [0,1]$.

If only the first two properties are satisfied and if H(a, 1) = a for all $a \in A$, then A is called a *deformation* retract of X. If there is just an $r: X \to A$ with $r \circ i = id$, then A is called a *retract*.

Back to linear algebra. Use the Gram-Schmidt process to show that $O(n) \subset GL_n(\mathbb{R})$ is a retract. Can you improve this retraction to a (strong) deformation retraction? (In order to simplify things, you could use the Gram-Schmidt process to prove a splitting $GL_n(\mathbb{R}) \cong O(n) \times B_n$ for some suitable B_n .)

(3 points)

8 Let C_* be an arbitrary chain complex. Is it true that the sequence of chain complexes

$$0 \to C_* \xrightarrow{p^{\bullet}} C_* \to C_*/pC_* \to 0$$

is always exact? (Proof or counterexample)

(1 point)

(3 points)

(5 points)