

Exercises in Algebraic Topology

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Summer term 2010

No 12

Due: Tuesday, 13th of July 2010

These exercises give extra points for those who need them. Solve them anyway, because they're important!

- 45** Let T^n be the n -torus, *i.e.*, the n -fold product of \mathbb{S}^1 . Calculate the cohomology ring $H^*(T^n)$. (4 points)
- 46** Show that there is an additive isomorphism $H^*(\mathbb{S}^2 \times \mathbb{S}^4) \cong H^*(\mathbb{C}P^3)$. Show however that the corresponding graded cohomology rings are *not* isomorphic. (3 points)
- 47** Let M be an odd dimensional manifold which is connected and compact but possibly non-orientable. Prove that the Euler characteristic is zero. (Here you can use what we already used without proof: $H_*(M)$ is finitely generated.) (2 points)
- 48** Let M be a connected compact 3-manifold. Show that $H_1(M)$ cannot be finite if M is non-orientable. (3 points)