Exercises in Algebraic Topology

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No 11 Due: Tuesday, 6th of July 2010

41 Let M be a connected orientable compact manifold of dimension m. Consider continuous maps $f\colon M \to \mathbb{S}^m$. Show that for every $k \in \mathbb{Z}$ there is a continuous map f as above whose degree is k. (Show the existence of a map of degree one first, and then use what you know about spheres.)

(3 points)

42 Let M and N be compact orientable connected manifolds of the same dimension and let $f\colon M \to N$ be a continuous map of degree one. Prove or disprove that $H_p(f)\colon H_p(M) \to H_p(N)$ is an epimorphism for all p.

(3 points)

43 Let M be a compact connected orientable 3-manifold. Express $H_1(M)$ as $\mathbb{Z}^n \oplus T$ where T is a finite abelian group. Calculate $H_2(M)$.

(2 points)

44 Let

$$\mathbb{R}^m_- := \{(x_1, \ldots, x_m), x_i \in \mathbb{R}, x_1 \leqslant 0\}$$

an m-dimensional half-space. The topological boundary of it is

$$\partial\mathbb{R}^m_- = \{(x_1, \ldots, x_m), x_i \in \mathbb{R}, x_1 = 0\}.$$

An m-dimensional topological manifold with boundary M is a hausdorff space with a countable basis of its topology such that there are homeomorphisms $h_i\colon U_i \to V_i$ with open subsets U_i in M and V_i in $\mathbb{R}^m_-$ and the U_i cover M. An $x \in M$ is called a boundary point, if there is a homeomorphism $h\colon U \to V$ with U open in M, V open in $\mathbb{R}^m_-$, $x \in U$ and $h(x) \in \partial\mathbb{R}^m_-$. The set of boundary points is called boundary of M, ∂M.

Prove that the following spaces are manifolds with boundary: $\mathbb{D}^2 \times \mathbb{S}^1$, $[0,1]$ and $(\mathbb{S}^1 \times \mathbb{S}^1)\setminus\mathring{\mathbb{D}}^2$, where $\mathring{\mathbb{D}}^2$ is suitably embedded in the torus.

What is the boundary?

(4 points)