Exercise Sheet 9 for Topological Infinite Graph Theory, Summer 2020
(to be discussed on 29. Juni 2020)

1. Show that t.f.a.e. for a locally finite connected graph G:
   (1) G is planar,
   (2) G contains no subdivision of K_5 and $K_{3,3}$,
   (3) $|G|$ contains no subspace homeomorphic to K_5 and $K_{3,3}$, and
   (4) $|G|$ is planar.

Let $x \neq y$ be vertices of a graph-like metrizable continuum (X, V, E). We say X is k-connected between x and y if the deletion of at most $k-1$ points from $X - \{x, y\}$ does not separate x from y. And X is k-connected if it is k-connected between all pairs of vertices.

2. Show that a 2-connected graph-like metrizable continuum X is planar if and only if X contains no subspace homeomorphic to K_5 or $K_{3,3}$.

3. Let $X = \lim_{\leftarrow} G_n$ be a 2-connected graph-like metrizable continuum. Must every G_n be 2-connected?

4.$^+$ Let $X = \lim_{\leftarrow} G_n$ be a graph-like metrizable continuum. Show that X is k-connected provided for any two vertices $x \neq y$, there are infinitely many G_n that are k-connected between $\pi_n(x)$ and $\pi_n(y)$. What about the converse implication?

Hinweise

1. For (2) $\Rightarrow$ (3) argue similarly to Lemma B. For (3) $\Rightarrow$ (4) build on Claytors Theorem.
2. You may use Claytor's theorem together with the fact that in a 2-connected graph-like metrizable continuum, every two points lie on a common simple closed curve.
3. Consider e.g. $|G|$ for a double ladder G.
4.