Exercise Sheet 8 for Topological Infinite Graph Theory, Summer 2020
(to be discussed on 22. Juni 2020)

Let $G = (V, E, \Omega)$ be a locally finite connected graph, $\mathcal{C}$ its topological cycle space, $\mathcal{B}$ its cut space, and $\mathcal{E} = \{0,1\}^E$ its edge space.

1.$^-$ Is the subspace $\mathcal{C}_{\text{fin}}$ of $\mathcal{C}$ consisting of all its finite elements the same as the space of finite sums of finite circuits?

2. Deduce Cor 8.7.4 from Theorem 8.7.1 without using Theorem 8.7.3.

3. Formulate and prove an inverse limit description of $\mathcal{C}$ and $\mathcal{B}$.

4. Prove directly, without using Lemma 2.2 (ii), that every set $F \subseteq E$ that meets every finite circuit in G evenly is a thin sum of fundamental cuts of any fixed ordinary spanning tree.

Let X be a metric graph-like continuum. The *cycle space* $\mathcal{C}(X)$ is defined as expected: all thin sums of edge sets of topological circles in X. The *topl. cut space* $\mathcal{B}_{top}(X)$ consists of all (finite) topological cuts $E(A, B)$ for (A, B) a clopen partition of $V(X)$.

5.$^-$ Show that $\mathcal{B}_{fin}(G) = \mathcal{B}_{top}(|G|)$.

6. Prove Theorem 8.7.1 for metric graph-like continua X by showing that TFAE for a set $D \subset E(X)$ of edges:
   (i) $D \in \mathcal{C}(X)$,
   (ii) D meets every topological cut of X evenly,
   (iii) D is a thin sum of fundamental circuits of any TST of X.

Hinweise

1. You might need to apply a theorem.
2. Generate every summand from fundamental circuits of the same topological spanning tree.
3. Let $S_n \subset V(G)$ and $G_n \preccurlyeq G$ as usual. What is the connection between $\mathcal{C}(G)$ and $\mathcal{C}(G_n)$ (for $n \in \mathbb{N}$) and what is the connection between $\mathcal{B}(G)$ and $\mathcal{B}(G[S_n])$?
4. Dualise the proof of Lemma 2.2 (ii) given in the lecture.
5.$^-$ Jumping Arc Lemma.
6. Follow Theorem 8.7.1.