Exercise Sheet 7 for Topological Infinite Graph Theory, Summer 2020
(to be discussed on 15. Juni 2020)

1. Suppose $X = \lim_{\leftarrow} G_n$ is a metrizable graph-like continuum with inverse limit representation, and let $\pi_n \colon X \to G_n$ be the projection map. Show that for any two disjoint closed $V_0, V_1 \subset V(X)$, there is $n \in \mathbb{N}$ with $\pi_n(V_0) \cap \pi_n(V_1) = \emptyset$.

2. Give two proofs for the fact that every metrizable graph-like continuum has a topological spanning tree:
   (1) By adapting the proof from the lectures of Lemma 8.6.11.
   (2) By considering an inverse limit representation $X = \lim_{\leftarrow} G_n$ and constructing a compatible family of spanning trees $T_n \subseteq G_n$.

3. Formulate and prove a version of the TST packing theorem for metrizable graph-like continua.

4. Formulate and prove a version of the topological Euler tour theorem for metrizable graph-like continua.

Bonus:

5.$^+$ Let X be a metrizable graph-like continuum and $x \neq y \in V(X)$. Prove the following Menger-type result: The maximal number of edge-disjoint $x - y$ arcs in X equals the minimum size of an $x - y$ cut.

Hinweise

1. Either find a finite edge cut $E(A, B)$ that separates V_0 from V_1. Or use an argument similar to the proof of Theorem 8.8.2 from the lectures where we showed that the map g is surjective.
2. For (2), don't forget to argue why $\lim_{\leftarrow} T_n$ cannot contain cycles.
3. Count cross edges across finite *clopen* partitions $\mathcal{P} = (U_1, \dots, U_{|\mathcal{P}|})$.
4. Compare with exercise Sheet 5 Q1. Apply the lifting lemma to save work.
5.+ Let $X = \lim_{\leftarrow} G_n$. Can you use that edge-Menger holds in G_n?