Exercise Sheet 3 for Topological Infinite Graph Theory, Summer 2020
(to be discussed on 11. May 2020)

All graphs in Q1 – Q4 are connected and locally finite.

1.$^{-}$ (i) Are fundamental cuts of ordinary spanning trees in fact bonds?
(ii) Are fundamental cuts of topological spanning trees in fact bonds?

2. Find a graph G with a connected standard subspace of $|G|$ that is the closure of a union of disjoint circles.

3. Prove or disprove that if a standard subspace of $|G|$ contains two vertex-disjoint arcs ending in an end ω it also contains two arcs ending in ω that are otherwise completely disjoint.

4. Every arc induces on its points a linear ordering inherited from $[0, 1]$. Call an arc in $|G|$ *wild* if it induces on some subset of its vertices the ordering of the rationals. Show that every arc containing uncountably many ends is wild.

5. Show that connected graphs (not necessarily locally finite) with only one end have topological spanning trees.

Bonus:

6.$^{++}$ Is it true that every connected graph (not necessarily locally finite) has a topological spanning tree? What about graphs with finitely many ends? With countably many? Arbitrarily many?

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1.$^{-}$ If you answer 'no', find a counterexample.

2. How many disjoint circles do you need so that their union can have a connected closure? Have you seen a connected standard subspace that is the closure of a union of disjoint arcs?

3. Solve the previous exercise first.

4. Show that every interval of the arc that contains uncountably many ends contains a vertex splitting it into two such intervals.

5. Start with a maximal set of disjoint rays.

6.$^{++}$