Exercise Sheet 10 for Topological Infinite Graph Theory, Summer 2020
(to be discussed on 6. July 2020)

1. Find a short proof that end spaces of connected (not necessarily locally finite) graphs are normal.

2. Find a short proof that the end space of a graph G is compact if and only if there is no finite vertex set $X \subseteq V(G)$ such that $G - X$ has infinitely many components containing a ray.

3. Conclude with the help of the previous exercise that $|G|$ with VTop is compact if and only if there is no finite vertex set $X \subseteq V(G)$ such that $G - X$ has infinitely many components.

4. An important cornerstone of the proof of the main theorem from the last lecture was the direction's theorem. Show that, in turn, the main theorem of the last lecture implies the direction's theorem.

5. Let G be any graph and $U \subseteq \Omega(G)$ an open set. Show that U is again the end space of a graph, i.e., there is a graph G' such that $\Omega(G')$ is homeomorphic to U.

Hinweise

1. Apply a theorem.
2. Apply a theorem.
3. Have you seen an operation that 'extends' results from $\Omega(G)$ to $|G|$
4.
5. To obtain G', find a suitable set of rays (in G) all belonging to ends in U.