1. Let $A, B$ be two vertex sets in a locally finite connected graph $G$. Can there be an infinite sequence $P_1, P_2, \ldots$ of disjoint $A - B$ paths such that each $P_{n+1}$ arises from $P_n$ by applying an alternating walk, and such that some edge $e \in G$ lies in $E[P_n]$ for infinitely many $n$ but not in $E[P_n]$ for infinitely many other $n$?

2. Prove the following strengthening of Lemma 3.3.2: If for a system $P$ of disjoint $A - B$-paths there is an alternating walk $W$ ending in $B \setminus V[P]$, then there also exists such an alternating walk $W'$ such that the symmetric difference $E[P] \triangle E(W')$ is precisely the edge-set of a system $P'$ of disjoint $A - B$-paths with $|P'| = |P| + 1$.

3. Let $G$ be a locally finite graph. Let us say that a finite set $S$ of vertices separates two ends $\omega$ and $\omega'$ if $C(S, \omega) \neq C(S, \omega')$. Use Proposition 8.4.1 to show that if $\omega$ can be separated from $\omega'$ by $k \in \mathbb{N}$ but no fewer vertices, then $G$ contains $k$ disjoint double rays each with one tail in $\omega$ and one in $\omega'$. Is the same true for all graphs that are not locally finite?

4. Prove the following more structural version of Exercise 2 on Sheet 8. Let $\omega$ be an end of a graph $G$. Show that either $G$ contains a $TK^8_0$ with all its rays in $\omega$, or there are disjoint finite sets $S_0, S_1, \ldots$ such that, if $C_i$ is the component of $G - (S_0 \cup S_i)$ that contains a tail of every ray in $\omega$, we have for all $i < j$ that $C_i \supseteq C_j$ and $G[S_i \cup C_i]$ contains $|S_i|$ disjoint $S_i - S_{i+1}$ paths for all $i \geq 1$.

5. Is there a planar $K_0$-regular graph all whose ends have infinite vertex-degree?
Hinweise

1. Try to construct $G$ together with the $P_{n}$.
3. Look at next exercise and its hint. For locally finite $G$ the sets $S_i'$ are very easy to find, and no normal spanning tree is needed.
4. Use a normal spanning tree to find provisional sets $S_1', S_2', \ldots$ of arbitrary finite cardinality that have the separation properties required of the $S_i$. Then use these to find the $S_i$.
5. Use the previous exercise.