Medieval Logic and Indian Logic: The Interactive and Epistemological Turn

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Modern Formalisms for Pre-Modern Indian Logic and Epistemology
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Plan of the talk

- Divisions of western logic.
- The interactive turn.
- Medieval logic.
- Indian logic.

"Traditional" vs. "Modern" Logic (1)

- "Traditional" logic:

- "Modern" logic:

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  - Aristotelian syllogistics.
- "Modern" logic:

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  - ▶ Classical logic à la Frege & Russell.
  - ▶ Mathematical logic based on set theory.

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  - ▶ (Modal logic/possible worlds semantics.)

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Focus on formal deduction.

Reference(s)

Pahi, B. 2008. "On relating two traditions of logic", in *Logic, Navya-Nyāya & Applications. Homage to Bimal Krishna Matilal*, ed. by M.K. Chakraborty, B. Löwe, M.N. Mitra, S. Sarukkal, (London: College Publications): 235–260.

"Traditional" vs. "Modern" Logic (2)

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  - ▶ Developed in the late 19th/early 20th century.
  - ▶ Based on the works of Frege, Russell/Whitehead.
- "Modern" logic:

"Traditional" vs. "Modern" Logic (2)

- "Traditional" logic:
  - ▸ Developed in the late 19th/early 20th century.
  - ▸ Based on the works of Frege, Russell/Whitehead.
- "Modern" logic:
  - ▸ Resurgence in last ∼30 years.
  - ▸ Focus on non-classical logics, applications beyond mathematics.

Traditional vs. Modern Logic (3)

- "Traditional" logic

- "Modern" logic

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- "Traditional" logic
  - ▶ Static
  - ▶ Universally applicable.
  - ▶ Grounded in mathematics.
- "Modern" logic

Traditional vs. Modern Logic (3)

- "Traditional" logic
  - ▶ Static
  - ▶ Universally applicable.
  - ▶ Grounded in mathematics.
- "Modern" logic
  - ▶ Dynamic.
  - ▶ Situationally applicable.
  - ▶ Grounded in language.

The Interactive Turn

- Dynamic systems designed for application to real world situations.
- Model knowledge and belief, interaction, and reasoning (in multi-agent systems).
- Emphasis on situational and applied aspects of logic.
- Context-dependent and pragmatic.

The Interactive Turn: examples

- Dynamic Epistemic Logic.
- Belief revision.
- Dialogical logic.
- Logics for negotiation.
- Logics for interactive computer systems.

Dynamic Epistemic Logic

Combination of

- Epistemic logic, with modal operators for agent's knowledge.
  - Can express things like "Bob knows that Alice doesn't know φ", "Everyone knows that everyone knows ...that φ".
- Dynamic logic, with programmes modeling actions.
  - Public announcement: After an agent a (truthfully) announces that φ, then any possibility where φ was false is removed from the model.

Reference(s)

van Ditmarsch, Hans, van der Hoek, Wiebe, & Barteld Kooi. 2007. *Dynamic Epistemic Logic*, Synthese Library Series, vol. 337, (Springer).

Belief revision

Agents change their beliefs on the basis of new information:

- Update: adds a new belief to the belief set.
- Revision: changes a belief by the addition of new belief which is possibly inconsistent with the belief set.
- Retraction: removes a belief from the belief set.

Reference(s)

Baltag, Alexandru, Moss, Lawrence S., & Slawomir Solecki. 1998. "The logic of public announcements, common knowledge, and private suspicions", in *Proceedings of the 7th conference on Theoretical Aspects of Rationality and Knowledge*: 43–56.

Dialogical logic

Proof is a dialogue or a game between two agents, the Proponent and the Opponent.

- Different rules for the agents determine different types of logic (e.g., classical logic, intuitionistic logic).
- A formula φ is valid iff the Proponent has a winning strategy for φ, that is, for any legal move of Opponent, Proponent always has a legal move, and eventually there are no more legal moves for Opponent.

Reference(s)

Lorenzen, P. & Kuno Lorenz. 1978. *Dialogische Logik* (Wissenschaftliche Buchgesellschaft).

Logics for negotiation

- Negotiation:
  - ▶ "Resource-sensitive" logics like linear logic rational negotiation.
  - ▶ Determining whether a deal (an exchange of resources) is rational reduces to a question of provability in linear logic.
  - ▶ Similarly for the question of whether some utility-maximizing sequence of deals exists.

Reference(s)

Porello, Daniele & Ulle Endriss. 2010. "Modeling multilateral negotiation in linear logic", in *Proceedings of the 19th European Conference on Artificial Intelligence*.

Logics for interactive computer systems

- Static logics can be used to model runs, i.e., sequences of actions, of computer programs.
- Dynamic logics can be used to model interaction between computer programmes and the environment (e.g., a user).
  - ▶ Uses a game-theoretic approach to logic, like dialogical logic: The computer is the Proponent, and the environment is the Opponent.
  - ▶ Can it be proved, of a specific programme, that it will always have a legal move, no matter what moves the environment makes?

Reference(s)

Alur, Rajeev, Henzinger, Thomas A., & Oma Kupfermann. 2002. "Alternating-time temporal logic", *Journal of the ACM* 49, no. 5: 672–713.

Where does medieval logic fit? (1)

- Concerned with techniques of reasoning that could be applied in real reasoning contexts.
- Dynamic, interactive systems, where reasoning is a dispute or debate between two or more agents.
- Varied from context to context.
- Pragmatic.
- Regimentation of natural language.

Logical modeling

- Using logical/formal techniques to build models of nonformal theories.
- Like mathematical modeling: Use the models to make 'predictions'.
- Importance of faithfulness.

Some examples

- Conditional necessity in Aristotelian modal syllogistics.
- Anselm's logic of agency.
- John Buridan's interval-based temporal semantics.
- Roger Swyneshed's solution to the Liar paradox.
- *Obligationes*: dialogical, dynamic, & epistemic aspects.

Conditional necessity in Aristotelian modal syllogistics

- Aristotelian modal syllogistics are often viewed as hopelessly muddled if not downright inconsistent.
- Rescher: modal syllogistics involve general laws and special cases.

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Definition

A syllogism $\mathcal{S}$ with special case s is valid for any simple model $\mathfrak{M}^{S}$ and $w \in W$ iff the following three conditions hold (i) $\mathfrak{M}^{S}, w \vDash M$, (ii) $\mathfrak{M}^{S}, w \vDash m$, and (iii) $\mathfrak{M}^{S} \upharpoonright s, w \vDash c$.

Note

This is sound and complete for the apodeictic fragment.

Reference(s)

Uckelman, Sara L. & Spencer Johnston. 2010. "A simple semantics for Aristotelian apodeictic syllogistics", *Advances in Modal Logic* vol. 8, forthcoming.

Anselm's logic of agency

- Drawn from material in the philosophical fragments (Lambeth MS 59)
- Demarcation of grammar and logic (cf. also *De grammatico*).
- Grammar is descriptive, logic is prescriptive.
- Relationship between agents and action.
- Interaction between agents.
- Can be modeled using neighborhood semantics.

Reference(s)

Uckelman, Sara L. 2009. "Anselm's logic of agency", *Logical Analysis and History of Philosophy* 12: 248–268.

John Buridan's interval temporal semantics

- "Now" is an interval, not a point.
- Pragmatics: the length of "now" is context-dependent.
- Upwards homogeneity vs. downwards homogeneity.

Reference(s)

Goranko, Valentin, Montanari, Angelo, & G. Sciavicco. 2004. "A road map of propositional interval temporal logics and duration calculi", *Journal of Applied Non-Classical Logics* 14, nos. 10–2: 11–56.

Uckelman, Sara L. & Spencer Johnston, "John Buridan's *Sophismata* and interval temporal semantics", *Logical Analysis and History of Philosophy* 13, forthcoming.

Roger Swyneshed's solution to the Liar paradox

- "A true sentence is a sentence that does not falsify itself and that principally signifies as is the case... A false sentence is an expression that falsifies itself, or else an expression that does not falsify itself and that principally signifies other than is the case".
- The Liar Paradox is self-falsifying.
- Notion of "immediately relevant to inferring that it itself is false" can be formalized with dynamic pointer semantics.

Reference(s)

Spade, Paul V. 1983. "Roger Swyneshed's theory of insolubilia: a study of some of his preliminary semantic notions", in *History of Semiotics*, A. Eschbach & J. Trabant, eds., Foundations of Semiotics 7, (Amsterdam: John Benjamins).

Uckelman, Sara L. & Benedikt Löwe. "Dynamic pointer semantics", in preparation.

Obligationes

- Two players, the **opponent** and the **respondent**.
- The **opponent** starts by positing a *positum* φ^*.
- The **respondent** can "admit" or "deny". If he denies, the game is over.

Obligationes

- Two players, the **opponent** and the **respondent**.
- The **opponent** starts by positing a *positum* φ^*.
- The **respondent** can "admit" or "deny". If he denies, the game is over.
- If he admits the *positum*, the game starts.
- In each round n, the **opponent** proposes a statement φ_n and the **respondent** either "concedes", "denies" or "doubts" this statement according to certain rules.
- He must concede propositions following from the *positum*, propositions already conceded, and the negations of propositions denied; he must deny propositions whose negation follows from this set.
- If φ neither follows from nor is repugnant to the foregoing, it is *irrelevant*, and he must concede it if it is true and deny it if it is false.

Dialogical aspects of *obligationes*

- King explains the apparent "content-freeness" of obligational disputations by pointing out that

  > they operate at a higher level of logical generality than that at which substantive debate occurs. If this is correct, then actual obligational moves—perhaps even recognized as such—are the vehicle whereby real argument takes place...they provide "meta-methodology" for reasoning.

- Different rules for different types of *obligationes* induce protocols for formal dialogue systems.

Reference(s)

King, Peter. 2004. "Opposing and responding: comments on Paul Spade", preprint, http://individual.utoronto.ca/pking/presentations/Spade_Comments.pdf

Uckelman, Sara L. "*Obligationes* as formal dialogue systems", in submission.

Dynamic aspects of *obligationes*

- In each turn of an *obligatio*, the Respondent announces his action: concede, deny, doubt. Following Burley's rules, announcements of concession and denial cause the set of relevant propositions to change.
- This (and many other types of rules) can be modeled with a type of Dynamic Epistemic Logic.
- Highlights the importance of the epistemic clauses (cf. later).

Reference(s)

Uckelman, Sara L. "A unified dynamic framework for modeling *obligationes*", in submission.

Uckelman, Sara L. "Deceit and nondefeasible knowledge: the case of *dubitatio*", in preparation; to be presented at LOFT 2010.

Where does medieval logic fit? (2)

- Medieval logic is much more like modern logic, in its interactive and epistemological guise, than it is like traditional logic.

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- Medieval logic is much more like modern logic, in its interactive and epistemological guise, than it is like traditional logic.
- Even the "traditional" part of medieval logic is non-traditional.

What about Indian logic?

Shared features

1. The role of knowledge.

2. Reflections of natural language ambiguities.

3. Truth conditions for basic statements.

4. Distinction between term negation and sentence negation.

The role of knowledge: Navya-Nyāya

Two of the three rules establishing the legitimacy of a *hetu*:

2 The ground must be present in at least one locus where the probandum is known to be present.

3 The ground must be absent in all loci where the probandum is known to be absent.

The role of knowledge: Medieval logic (1)

Fundamental importance of *known* truth as opposed to simple (and potentially unknown) *truth* in *obligationes*.

Reference(s)

de Rijk, Lambertus Maria. 1975. "Some thirteenth century tracts on the game of obligation II", *Vivarium* 13, no. 1: 22–54.

The role of knowledge: Medieval logic (1)

Fundamental importance of *known* truth as opposed to simple (and potentially unknown) *truth* in *obligationes*.

- Early versions were purely truth-functional:
  - ▶ *Obligationes Parisienses*:
    - ★ "Everything which is put forward that has the same form of speech as what was first put down [the *positum*], everything following from the *positum* and a thing or things conceded [previously] and everything which is true and not repugnant to these must be conceded."
    - ★ "The opposite of the *positum* and every false thing not following from the *positum* and a thing or things conceded and the opposite or opposites of a thing or things correctly denied and every true thing repugnant to these must be denied".

Reference(s)

de Rijk, Lambertus Maria. 1975. "Some thirteenth century tracts on the game of obligation II", *Vivarium* 13, no. 1: 22–54.

The role of knowledge: Medieval logic (2)

- Later authors introduced epistemic clauses:
  - ► Richard Brinkley (late 14th C):
    - ★ "Everything following from the *positum* during the time of its positing and known to be such must be conceded."
    - ★ "Everything repugnant to the *positum* during the time of its positing and known to be such must be denied."
  - ► Other examples include William of Ockham, Paul of Venice, Peter of Mantua, . . .

Reference(s)

Brinkley, Richard, Spade, Paul V., ed., & G.A. Wilson, ed. 1995. *Richard Brinkley's Obligationes: A Late Fourteenth Century Treatise on the Logic of Disputation*, Beiträge zur Geschichte der Philosophie und Theologie des Mittelalters, (Aschendorff).

Modeling natural language

Both Latin and Sanskrit

- lack definite and indefinite articles.
- sometimes lack explicit quantifiers.

Syntactic ambiguity: Navya-Nyāya

ghaṭo nīlaḥ
pot [is] blue

can mean "the pot is blue", "some pot is blue", or "every pot is blue".

Syntactic ambiguity : Latin

homo est albus

can mean "the man is white", "some man is white", or "man is white".

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homo est animal

can mean "the man is animal" (adjectival), "the man is an animal" (nominal), "some man is animal", and so on.

Regimentation of natural language: Navya-Nyāya

Both traditions solved the problem of ambiguity by regimentation of natural language.

- Naiyāyikas: two solutions to the interpretation of an unquantified subject in a subject + predicate sentence:
  - ▸ Meaning Particularism (*vyaktiśaktivāda*): the unquantified subject is understood as being existentially quantified and has "genuine referring use".
  - ▸ Meaning Universalism (*jātiśaktivāada*): the unquantified subject is understood in a universal sense, like Russell's treatment of definite descriptions.

Regimentation of natural language: western tradition

- Medieval western logicians: four types of subject + predicate sentences, on the basis of their quantity: universal, particular, singular, indefinite:
  - ▶ Common view (e.g., William of Sherwood): Sentences such as the examples above would count as indefinite.
  - ▶ Peter Abelard: The simple affirmative statement 'A human being is white' [*homo est albus*] should be analysed as claiming that that which is a human being is the same as that which is white (*idem quod est homo esse id quod album est*).

Reference(s)

Knuuttila, Simo. 2007. "How theological problems influenced the development of medieval logic?" in *"Ad Ingenii Acuitionem": Studies in Honour of Alfonso Maierù*, ed. by S. Caroti, R. Imbach, Z. Kaluza, G. Stabile, & L. Sturlese, (Louvain-la-Neuve: Fédération Internationale des Instituts d'Études Médiévales).

William of Sherwood. 1966. *William of Sherwood's Introduction to logic*, trans. by N. Kretzmann, (Minneapolis, MN: University of Minnesota Press).

Truth conditions for predications

Ganeri's "translation manual": Navya-Nyāya → predicate logic.

Note

Every sentence in the language (NN*) has the structure

$$(-)(\forall/\exists)(\exists)(-)(_{\sqcup 1} R_{\sqcup 2})$$

Reference(s)

Ganeri, Jonardon. 2008. "Towards a formal regimentation of the Navya-Nyāya technical language I", in *Logic, Navya-Nyāya & Applications. Homage to Bimal Krishna Matilal*, ed. by M.K. Chakraborty, B. Löwe, M.N. Mitra, S. Sarukkal, (London: College Publications): 105–121.

Ganeri, Jonardon. 2008. "Towards a formal regimentation of the Navya-Nyāya technical language II", in *Logic, Navya-Nyāya & Applications. Homage to Bimal Krishna Matilal*, ed. by M.K. Chakraborty, B. Löwe, M.N. Mitra, S. Sarukkal, (London: College Publications): 123–138.

Truth conditions for categorical propositions

If we replace R with E (for Latin *est*), Ganeri's formalization exactly matches the truth conditions given by Peter Abelard.

- Indefinite statements: In *homo est albus* we read 'that which is a human being' universally and 'that which is white' particularly.

- Definite statements: simply replace 'that' with the relevant quantifier (everything, something, nothing).

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$$
\begin{array}{ll}
\text{A:} & \forall x \exists y (xEy) \\
\text{E:} & \forall x \forall y \neg (xEy) \\
\text{I:} & \exists x \exists y (xEy) \\
\text{O:} & \exists x \forall y \neg (xEy)
\end{array}
$$

Restricted quantification

Both traditions usually use restricted quantifiers, e.g., (in Ganeri's notation):

$$(-)(\forall/\exists : \tau)(\exists : \pi)(-)(\sqcup_1 R \sqcup_2)$$

Term negation vs. sentence negation

- Both are term logics (as opposed to predicate or propositional logics).
- Two types of negation: term (internal) negation and propositional (external) negation.
- Term-negation: "absence of a property is itself a property".
- They differ in which type of negation is considered primary.

The relationship between grammar and logic

The delineation of grammar and logic is important in both traditions.
However, the importance is assigned differently.

- Medieval tradition:
  - ▶ Anselm: the job of the grammarian is descriptive, the job of the logician is prescriptive.
  - ▶ Modist Grammarians: developed speculative grammar "to systematise a universal semantic approach to language, leading to a high degree of sophistication and adequacy in linguistic description".
  - ▶ Grammar for the Modistae is a *derivatively prescriptive* science.
- Indian tradition:

  a system of grammar for a language aims at characterizing the informal notion of grammatical correctness of words and sentences for the language in question... The grammarian is not at liberty to alter the boundary between admissible and inadmissible data. Grammar is descriptive subject only to minimal constraints on the admissibility of data

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Reference(s)

Pinborg, Jan. 1982. "Speculative grammar", *The Cambridge History of Later Medieval Philosophy*, ed. by N. Kretzmann, A. Kenny, J. Pinborg, (Cambridge: Cambridge University Press): 254–269.

Conclusions

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Conclusions

- There are many similarities between Navya-Nyāya and medieval western logic.
- These arise from their shared focus on the applied, pragmatic aspects of reasoning and rationality.
- Modern dialogical and epistemological approaches to logic have shown themselves well-suited for modeling medieval Western logic.
- Therefore these particular tools are probably well-suited for modeling Indian logic as well.