Proposition 1.26. Any grammar is equivalent to an ε-adequate grammar. Moreover, any grammar with property Q is equivalent to an ε-adequate grammar with property Q.

Proof. Suppose $G = (\Sigma, V, P, S)$ is a grammar. Take a new variable $T \notin V$ and let

$$\begin{aligned} V' &:= V \cup \{T\}, \\ P' &:= P \cup \{T \to \alpha\,;\, S \to \alpha \in P\}, \text{ and} \\ G' &:= (\Sigma, V', P', T). \end{aligned}$$

Clearly, all rules in P' are T-safe, so G' is ε-adequate and obviously $\mathcal{L}(G) = \mathcal{L}(G')$. Observe that the transformation $P \mapsto P'$ preserves all four properties that Q can stand for. Q.E.D.