Proposition 1.24. Let G and G' be grammars that do not share any variables and are variable-based. Let H be their union grammar. Then $\mathcal{L}(H) = \mathcal{L}(G) \cup \mathcal{L}(G')$.

Proof. Clearly, if $S \xrightarrow{G} v$, then $T \xrightarrow{H} v$ by using the rule $T \to S$; similarly, if $S' \xrightarrow{G'} v$, then $T \xrightarrow{H} v$. Thus, $\mathcal{L}(G) \cup \mathcal{L}(G') \subseteq \mathcal{L}(H)$.

Since $V \cap V' = \varnothing$ and the grammars are variable-based, no rule from P can apply to a string that contains no variables from V and no rule from P' can apply to a string that contains no variables from V'. As a consequence, we see (by induction) that any H-derivation starting from S will only use rules from P and any H-derivation starting from S' will only use rules from P'. Thus, if $S \xrightarrow{H} v$, then $S \xrightarrow{G} v$ and if $S' \xrightarrow{H} v$, then $S' \xrightarrow{G'} v$. But since there are only two rules involving T, any H-derivation $(\sigma_0, \sigma_1, \ldots, \sigma_n)$ with $\sigma_0 = T$ will have $\sigma_1 = S$ or $\sigma_1 = S'$, so $T \xrightarrow{H} v$ implies either $S \xrightarrow{G} v$ or $S' \xrightarrow{G'} v$. Q.E.D.