Example 1.12. As an illustration, we show that Chomsky's example "Furiously sleep ideas green colourless" is not derivable in the grammar given in § 1.3. We first need to specify the grammar G formally: Σ is the finite set of all words in some English dictionary,

$$V := \{S, \mathrm{NP}, \mathrm{VP}, \mathrm{Adj}, \mathrm{Adv}, \mathrm{Verb}, \mathrm{Noun}\},$$

and P is the list of production rules given in (*) together with the dictionary rules that transform the nonterminals into the corresponding terminals, i.e., the relevant production rules are

$$\begin{aligned} S &\to \mathrm{NP}\,\mathrm{VP}, & \mathrm{Noun} &\to \mathrm{ideas},\\ \mathrm{NP} &\to \mathrm{Adj}\,\mathrm{NP}, & \mathrm{Verb} &\to \mathrm{sleep},\\ \mathrm{NP} &\to \mathrm{Noun}, & \mathrm{Adj} &\to \mathrm{colourless},\\ \mathrm{VP} &\to \mathrm{Verb}, & \mathrm{Adj} &\to \mathrm{green},\\ \mathrm{VP} &\to \mathrm{VP}\,\mathrm{Adv}, & \mathrm{Adv} &\to \mathrm{furiously}. \end{aligned}$$

We claim that no derivable string ends in either Adj or colourless (let's call a string that does not end in either of these two *good*) and show this by induction on the length of the derivation. Clearly, any derivation of length zero produces S which is a good string. Suppose all derivations of length n produce only good strings and assume that α is produced by a derivation of length $n+1$. Let's assume that the last step of that derivation is $\beta \xrightarrow{G}_1 \alpha$. Clearly, β has a derivation of length n, so by induction hypothesis, β is a good string.

An inspection of our grammar rules show that since β does not end in Adj and $\beta \xrightarrow{G}_1 \alpha$, then α does not end in Adj. So, if α is not good, it must end in colourless. Furthermore, the only rule that could produce colourless is the rule Adj $\to$ colourless. Thus, if α ends in colourless, then β must end in Adj. Contradiction!