We fix any set X. If n is a natural number (note that we include 0 in the natural numbers), then X^n is the set of n-tuples of elements of X; we call these objects *X-strings of length n* (usually denoted by letters such as α, β, γ, σ, and τ). In the usual set-theoretic representation, $n = \{0, 1, ..., n-1\}$ and a string of length n is a function from the set n into X. Note that X^0 only contains the empty sequence which we shall denote by ε. We write X^* for the set of all X-strings[1] and write $|\alpha| = n$ if $\alpha \in X^n$ (or equivalently, $\mathrm{dom}(\alpha) = n = \{0, ..., n-1\}$); the number $|\alpha|$ is called the *length of* α. Since strings are functions, we can use the usual notation for function restriction to denote their initial segments, i.e., if $\alpha \in X^n$ and $k \leq n$, then $\alpha{\restriction}k$ is the unique initial segment of α of length k.

If $\alpha, \beta \in X^*$, we can concatenate them in the usual way and write $\alpha\beta$ for the concatenated string. If α has length n and β has length m, then $\alpha\beta$ has length $n + m$:

$$\alpha\beta(k) := \begin{cases} \alpha(k) & \text{if } k < n \text{ and} \\ \beta(\ell) & \text{if } k = n + \ell \text{ and } \ell < m. \end{cases}$$

If $x \in X$, we use the notation x^n for the string of length n consisting only of the symbol x. Similarly, if $\alpha \in X^*$, we write α^n for the concatenation of n copies of the string α (formally, we can define this by recursion as $\alpha^0 := \varepsilon$, $\alpha^{n+1} := \alpha^n\alpha$). We often (slightly incorrectly) confuse $x \in X$ with the string of length 1 consisting of the element x. So, if we write αx, we mean the string α with an extra element x appended at the end; if we write $x\alpha$, we mean the string α prefixed by an element x. If $Y, Z \subseteq X^*$, we write $YZ := \{\alpha\beta\,;\, \alpha \in Y \text{ and } \beta \in Z\}$; if $Y = \{\alpha\}$, we abbreviate this to αZ and if $Z = \{\beta\}$, we write $Y\beta$.

Given any function $f : X \to Y$, we can recursively extend it to a function $\widehat{f} : X^* \to Y^*$ by

$$\widehat{f}(\varepsilon) := \varepsilon,$$
$$\widehat{f}(\alpha x) := \widehat{f}(\alpha) f(x) \text{ (for } \alpha \in X^* \text{ and } x \in X).$$