

**Proposition 1.2** (Cantor's Theorem). If  $X$  is infinite, then the power set of  $X$ , i.e., the set of all subsets of  $X$ , denoted by  $\wp(X)$ , is uncountable.

*Proof.* Let  $i : \mathbb{N} \rightarrow X$  be an injection. Suppose that  $\pi : \mathbb{N} \rightarrow \wp(X)$  is a function. We shall show that it is not a surjection. Define  $D := \{i(n) ; i(n) \notin \pi(n)\}$  and claim that  $D$  is not in the range of  $\pi$ . Assume otherwise, then there is some  $d \in \mathbb{N}$  such that  $D = \pi(d)$ . But then  $i(d) \in D = \pi(d)$  if and only if  $i(d) \notin \pi(d)$ . Contradiction! Q.E.D.