Common knowledge in conversation

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Df.1. The proposition p is common knowledge (CK) among the members of C iff

$$\forall x \in C \, \Box_x p,$$

$$\forall x \in C \, \forall y \in C \, \Box_x \Box_y \; p,$$

$$\forall x \in C \, \forall y \in C \, \forall z \in C \, \Box_x \Box_y \Box_z \; p$$

etc. ad infinitum ■

(thus Schiffer 1972 and epistemic logics)

Hypothesis: CK indispensable for communication

In order for me to felicitously utter p, its presuppositions must be CK among my interlocutors. (Lewis 1969)

In other words: for the members of C to communicate in language L, the meaning postulates of L must be common knowledge among them.

Otherwise always a risk that an utterance would not be felicitous – by domino effect over $\Box$-depth.

Paradox: CK computationally impossible (Clark and Marshall 1981)

In order to be sure that my utterance of p will be felicitous, I must check that p's presuppositions are CK.

But, by definition of CK, that amounts to checking an infinite number of sentences.

That cannot be done in a finite time, so I can never be sure that I'll utter p felicitously.

Yet usually we are quite certain about the felicity of what we are going to say. A contradiction.

Df.2. p is CK among the members of C at the world w_0 iff

$$\forall w \; R^*(w_0, w) \rightarrow w \models p$$

where R^* is the transitive closure of accessibility relations of all the members of C. ■

Hence p is CK iff p is true at all non-solitary worlds. If there are finitely many of those, the paradox is solved.

Nevertheless, this is cognitively implausible – because we have to scan the entire model.

Df.3. p is CK among the members of C iff a basis B exists s.t.

$$\forall x \in C \ \forall p \in B \ \Box_x p$$

where

$$B \models (p \wedge \forall x \in C \ \forall p \in B \ \Box_x p).$$

■ (Lewis 1962, Aumann 1976)

Psychologically more realistic – B a shared basis (Clark 1996). Enough to give a B to check CK.

However, this is non-categorical: unintended B-s are possible. Thus it is impossible to check finitely whether p is *not* CK.

Df.4. p is CK among the members of C if q obtains s.t.:

$$q \to \forall x \in C\, \Box_x (p \wedge q).$$

■ (Barwise 1989)

Such q describes a coordination device, e.g. the presence of a salient object in the common visual field – or a linguistic convention.

However, checking whether p is CK proceeds as for Df.2. – for the negative case the entire model must be scanned.

But: from the psychological point of view, there are finitely many coordination devices. So actually the search space is limited.

Question: how to restrict the set of eligible q-s?

Coordination through evolution

An answer: by saying what is the strategy behind treating coordination devices as sources of CK. Then the set of q-s will be delimited by the availability of such strategy.

Assume I am talking with b, so $C = \{b, \mathrm{me}\}$. Let action $\mathbf{p}$ be uttering the presupposition p; let $\alpha, \beta, \gamma \in \mathbb{R}_+$ be utilities. We face a decision problem:

	$\Box_b p$	$\neg\Box_b p$
$\mathbf{p}$	$-\alpha$	β
$\bar{\mathbf{p}}$	β	$-\gamma$

Cf. the quantity principle.

	$\Box_b p$	$\neg\Box_b p$
$\mathbf{p}$	$-\alpha$	β
$\bar{\mathbf{p}}$	β	$-\gamma$

Let EU – expected utility, P – probability:

$$EU(\mathbf{p}) = P(\Box_b p) \cdot -\alpha + (1 - P(\Box_b p)) \cdot \beta \; = \; \beta - P(\Box_b p)(\beta + \alpha)$$

$$EU(\bar{\mathbf{p}}) = P(\Box_b p) \cdot \beta + (1 - P(\Box_b p)) \cdot \gamma \; = \; P(\Box_b p)(\beta + \gamma) - \gamma$$

Thus in the long run if I am guessing about what b might know, the optimal strategy will be mixed:

$$EU(\mathbf{p}) > EU(\bar{\mathbf{p}}) \quad \text{iff} \quad P(\Box_b p) < \frac{\beta + \gamma}{2\beta + \gamma + \alpha}$$

Call it $\mathbf{m}$.

But my interlocutor b is in a symmetric situation. (A strong, but plausible assumption.) So she will also use $\mathbf{m}$; using some other $\bar{\mathbf{m}}$ must yield smaller EU, so for $\delta > \eta$:

	$\mathbf{m}$	$\bar{\mathbf{m}}$
$\mathbf{m}$	δ, δ	δ, η
$\bar{\mathbf{m}}$	η, η	η, δ

Summing the utilities for both me and b, we have a common utility from C's group perspective. Then:

$$U(\mathbf{m}, \bar{\mathbf{m}}) \leq U(\mathbf{m}, \mathbf{m})$$

$$U(\bar{\mathbf{m}}, \mathbf{m}) = U(\mathbf{m}, \mathbf{m}) \rightarrow U(\bar{\mathbf{m}}, \bar{\mathbf{m}}) < U(\mathbf{m}, \bar{\mathbf{m}})$$

So $\mathbf{m}$ is an evolutionarily stable strategy (ESS).

If one can use $\mathbf{m}$, one will use it; converse trivially. One can use it iff one is good at guessing the probabilities of what others in C know. So whoever uses $\mathbf{m}$ is good at guessing that – which amounts to using coordination devices.

Df.5. p is CK among the members of C if q obtains s.t.:

$$q \to \forall x \in C \, \Box_x (p \wedge q).$$

where q holds because of an ESS employed by the members of C. ■

This restricts the search space while checking CK to the set of ESS-s used in C, which is finite and an empirical question.