Foundations of the Formal Sciences
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May 9th, 15:30

Inner Models

The obvious method to survey possibilities is to design and examine models. In mathematics, an enormous number of mathematical models or structures have been studied as examples and counterexamples in order to gain insights into the strengths and limitations of initial assumptions. Geometrical models were constructed to show that the famous parallel postulate of Euclid cannot be decided on the basis of the other geometrical axioms.

Foundational studies in mathematics consider the possible behaviour of all mathematical objects; these may be identified with sets. Models of set theory have to be constructed inside the class of mathematical objects and are of necessity inner models (in a non–technical sense). Although most ordinary mathematical questions can be answered by the accepted set theoretical axioms the study of models of set theory has confirmed that there are natural questions about infinitary objects which are not decidable. K. Gödel constructed the inner model **L** of all constructible sets in which the continuum hypothesis of G. Cantor holds true. There are other models, in which this hypothesis fails. These results certainly have epistomological consequences.

My contribution will introduce the relevant notions of the axiomatic method by first discussing the "Grundlagen der Geometrie" by D. Hilbert which was first published exactly 100 years ago. I shall consider inner models in the geometrical context and proceed to axiomatic set theory. Gödel's model of constructible sets is the original paradigm of a whole spectrum of inner models which is at the centre of present day mathematical research.