Foundations of the Formal Sciences
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On Automated Higher-Order Theorem Proving and Henkin Completeness

It is a well known consequence of Gödels first incompleteness theorem that there can be no complete calculi for higher-order logic with respect to standard semantics. On the other hand Henkin has shown in [Hen50] that there are indeed complete calculi if one gives up the intuitive requirement of full function domains in standard semantics and considers so called Henkin models instead. In the first part of this paper we will sketch some well known approaches to automated theorem proving in classical higher-order logic (based on Church's simply typed λ-calculus) and discuss them with respect to Henkin completeness. Among these are traditional ones such Andrews' higher-order resolution approach [And71], the calculus of Jensen and Pietrzykowski [JP72] and Huet's constraint resolutionapproach [Hue72] but also some more recent ones like the E-resolution approach of Wolfram [Wol93] or Kohlhase's sorted variant [Koh94] of Huet's original approach. Common to all these approaches is that additional extensionality axioms are needed in order to ensure Henkin completeness.

In the second part of the paper we sketch the rather new extensional higher-order resolution approach $\mathcal{ER}$ [BK98] which is the first machine oriented Henkin complete calculus known to the author that does not require additional extensionality axioms. Instead of adding axioms to the search space this calculus additionally employs more goal oriented extensionality rules which realize a close interplay between higher-order (pre)unification and the overall proof search. We furthermore dicuss two extensions to $\mathcal{ER}$ that treat equality as a primitive notion: extensional higher-order paramodulation and RUE-resolution. The former is a typical representative for a term-rewriting approach and the latter for a difference-reducing approach.

Completeness proofs with respect to Henkin semantics are rather complicate and hardly manageable from scratch. In the last part of the paper we will therefore point to the abstract consistency method as an important tool for this purpose. This proof method was first introduced by Smullyan [Smu63] for first-order logic and later extended to higher-order logic by Peter Andrews [And71]. A recentextension to the latter that ensures Henkin completeness is provided by [BK97].

References

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