Understanding & Explanation Task U3

Deadline for Understanding & Explanation Task U3: Monday, 25 October 2021, 2pm. Please hand in via the `elo` webpage as a single pdf file.

*Understanding & Explanation tasks (**U**).*

Marking Scheme.

An answer will be considered **good** if all three criteria are satisfied. These answers will get full points (i.e., $3\frac{1}{2}$ **points**).

It will be considered **satisfactory** if it has minor deficiencies in some of the three criteria. Satisfactory answers will get **3 points**.

It will be considered **unsatisfactory** if it has a major deficiency in either correctness or comprehensivity. Unsatisfactory answers will get either **2 points**, **1 point**, or **0 points**, depending on the flaws.

Task U3. Work in ZFC. Let α be an infinite ordinal and consider the following two properties:

$\Phi_1(\alpha)$**:** The ordinal α is a countable union of sets that are not in bijection with α (i.e., $\alpha = \bigcup X$ where X is countable and for each $x \in X$, $x \not\sim \alpha$).

$\Phi_2(\alpha)$**:** The ordinal α is a countable union of sets that are proper subsets of α (i.e., $\alpha = \bigcup X$ where X is countable and for each $x \in X$, $x \subsetneq \alpha$).

Explain that, in general, Φ_1 is stronger than Φ_2 by arguing that Φ_1 implies Φ_2 but not vice versa.