Understanding & Explanation Task U1

Deadline for Understanding & Explanation Task U1: Monday, 27 September 2021, 2pm. Please hand in via the `elo` webpage as a single pdf file.

Understanding & Explanation tasks (U). Every two weeks, we shall have one question that asks students to explain in their own words a phenomenon encountered in lectures. In total, there will be six **U** sheets (**U1** to **U6**).

Understanding & Explanation tasks ask you to describe either mathematical concepts or mathematical proofs in your own words. These tasks correspond to the exam questions in **Part I** of the exam which is worth 7 out of 10 points and will decide whether you pass or fail the exam component. **Part I** of the exam will have two questions, each worth $3\frac{1}{2}$ points, therefore we also give $3\frac{1}{2}$ points for each of the **U** tasks. Your answer will be marked according to whether it is **correct**, **comprehensive**, and **well-structured**. An answer is *comprehensive* if all of the important mathematical ideas are discussed and explained. In the exam, if both of your answers in Part I are marked as **satisfactory**, you are guaranteed to get a passing exam mark.

An answer will be considered **good** if all three criteria are satisfied. These answers will get full points (i.e., $3\frac{1}{2}$ **points**).

It will be considered **satisfactory** if it has minor deficiencies in some of the three criteria. E.g., fixable errors in definitions or arguments would be considered a minor deficiency in correctness, the omission of one among several ideas or proof steps would be considered a deficiency in comprehensivity, a general lack of structure or confused prose would be considered a deficiency in being well-structured. Satisfactory answers will get **3 points**.

It will be considered **unsatisfactory** if it has a major deficiency in either correctness or comprehensivity, e.g., a flaw in a definition that invalidates the argument, a major error in an argument, or omitting the main idea of the proof would be considered major deficiencies. Unsatisfactory answers will get either **2 points**, **1 point**, or **0 points**, depending on the flaws.

Task U1. Consider the formula

$$\Phi(x) :\iff \exists y(y \in x) \wedge \forall y \forall z(y \in x \wedge z \in x \to y = z)$$

describing that x has exactly one element and the sentence

$$\sigma := \forall x \big(\Phi(x) \to \exists y \exists z \, (y \neq z \wedge y \subseteq x \wedge z \subseteq x) \big)$$

expressing that every one-element set has two distinct subsets. Explain how one can show that very weak axiom systems of set theory cannot prove σ. Also explain which axioms are sufficient to prove σ. Give concrete examples of axioms systems with these properties as well as proof sketches of these facts. (No detailed proofs are necessary and you are allowed to refer to the lecture notes in your arguments.)