Homework Sheet #5

MasterMath: Set Theory
2021/22: 1st Semester
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Deadline for Homework Set #5: Monday, 18 October 2021, 2pm. Please hand in via the `elo` webpage as a single pdf file.

(17) Suppose α and β are ordinals and define $F(\beta, \alpha) := \{f : \beta \to \alpha\,;\text{ for all but finitely many } \gamma \in \beta,\ f(\gamma) = 0\}$ Define an order $\prec$ on $F(\beta, \alpha)$ by

$$f \prec g :\iff f(\mu) < g(\mu) \text{ where } \mu := \max\{\gamma \in \beta\,;\, f(\gamma) \neq g(\gamma)\}.$$

Show that $(F(\beta, \alpha), \prec) \cong (\alpha^\beta, \in)$.

(18) Let x and y be elements of the cumulative hierarchy (therefore, their Mirimanoff rank is defined) and let $\varrho(x) = \alpha$ and $\varrho(y) = \beta$. Determine the Mirimanoff rank of $\mathrm{P}(x)$, $\bigcup x$, (x, y), and $x \times y$. Prove your claims.

(19) Let α be an ordinal. Show that $(\mathbf{V}_\alpha, \in)$ is a model of the axioms of Extensionality and Foundation.

(20) Let α be an ordinal. Show that the following three statements are equivalent:

- (i) For all $\beta < \alpha$, there is no bijection between α and β.
- (ii) For all $\beta < \alpha$, there is no injection from α into β.
- (iii) For all $\beta < \alpha$, there is no surjection from β onto α.