HOMEWORK SHEET #1

Deadline for Homework Set #1: Monday, 20 September 2021, 2pm. Please hand in via the `elo` webpage as a single pdf file.

(1) Consider the natural numbers $\mathbb{N}$ as a set of vertices in a graph with the edge relation E defined by $n\,E\,m$ if and only if $n < m$. Check whether the axioms of extensionality, pairing, union, power set and the axiom scheme of separation hold in the structure $(\mathbb{N}, E)$.

(2) Let $\mathbf{G} = (V, E)$ be the following graph model:
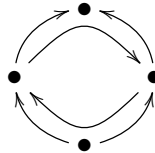
Check whether the axioms of extensionality, pairing, union, power set and the axiom scheme of separation hold in $\mathbf{G}$.

(3) Find a finite directed graph $\mathbf{G} = (V, E)$ that satisfies the axioms of extensionality, pairing, power set, and union. By a theorem from class, it cannot satisfy the axiom scheme of separation. Give a concrete instance of separation that fails in your graph.

(4) Consider the following *axiom of binary unions*:

$$\forall x \forall y \exists u \forall z (z \in u \leftrightarrow (z \in x \lor z \in y)).$$

Show that every graph that satisfies the axioms of pairing and union also satisfies the axiom of binary union.