Homework Sheet #1

Capita Selecta: Set Theory
2020/21: 1st Semester; block 1
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Homework. Homework is due on Fridays. Please submit your work as a single pdf file via Canvas. You will receive one point for each question that you attempt, independent of your performance on the question. The homework will not be formally marked, but the lecturers may give individual feedback in the case of problems.

Deadline. This homework set is due on **Friday, 11 September 2020, 1pm**.

1. Let M be a move set and $A, B \subseteq M^\omega$ are disjoint, we define the winning conditions for the game $\mathrm{G}(A, B)$ as follows: if the play of the game is $x \in M^\omega$, then player I wins if $x \in A$, player II wins if $x \in B$, and otherwise the game is a *draw*. A strategy σ is *winning or drawing in* $\mathrm{G}(A, B)$ *for player I* if for all strategies τ, we have that $\sigma * \tau \in A$ or $\sigma * \tau \notin B$, respectively. A strategy τ is *winning or drawing in* $\mathrm{G}(A, B)$ *for player II* if for all strategies σ, we have that $\sigma * \tau \in B$ or $\sigma * \tau \notin A$, respectively.

   Show that if A and $M^\omega \backslash B$ are determined, then at least one of the two players has a drawing strategy in $\mathrm{G}(A, B)$ and that both players have a drawing strategy if and only if none of them has a winning strategy.

2. Do Exercise 19.7 in Andretta's book draft on p. 218. (Note that Andretta's definition of equivalence of games is in Definition 19.6.)

3. Read Section 1.C.1 of Andretta's book draft (pp. 22–24); you can ignore the references to DC for now; you can read the direction "(a)⇒(b)" in the proof of Proposition 1.32 under the assumption of full AC and ignore the direction "(b)⇒(a)") and do Exercises 1.47, 1.48, and 1.49 (p. 27).