Homework Sheet #6

MasterMath: Set Theory
2020/21: 1st Semester
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Deadline for Homework Set #6: Monday, 19 October 2020, 2pm. Please hand in via the `elo` webpage as a single pdf file.

(18) Let X be a set of pairwise disjoint non-empty sets, i.e., if $x, x' \in X$, then $x \neq \varnothing \neq x'$ and $x \cap x' = \varnothing$. We say that C is a *choice set for* X if for each $x \in X$, the set $x \cap C$ has exactly one element. The *Axiom of Choice Sets* says that every set of pairwise disjoint, non-empty sets has a choice set.

Show that (on the basis of the axioms of ZF), the Axiom of Choice and the Axiom of Choice Sets are equivalent.

Why can't you get rid of the requirement that the sets in X are pairwise disjoint?

(19) Let $(W, <)$ be a wellorder. Define $\prec$ on $\mathrm{P}(W) \times \mathrm{P}(W)$ by $X \prec Y$ if and only if $X \neq Y$ and the $<$-smallest element of $(X \cup Y) \backslash (X \cap Y)$ is in X. Prove that $(\mathrm{P}(W), \prec)$ is a linear order.

(20) Let κ, λ, and μ be cardinals. Show the following rules of *cardinal arithmetic* by providing explicit bijections between the corresponding sets:

(a) $(\kappa \cdot \lambda)^\mu = \kappa^\mu \cdot \lambda^\mu$,

(b) $\kappa^\lambda \cdot \kappa^\mu = \kappa^{\lambda+\mu}$, and

(c) $(\kappa^\lambda)^\mu = \kappa^{\lambda \cdot \mu}$.