Homework Sheet #2

MasterMath: Set Theory 2020/21: 1st Semester K. P. Hart, Benedikt Löwe, Ezra Schoen, & Ned Wontner

Deadline for Homework Set #2: Monday, 21 September 2020, 2pm. Please hand in via the elo webpage as a single pdf file.

(5) Prove that for any two natural numbers n and m (i.e., elements of the smallest inductive set \mathbb{N}) one of the following holds:

 $n \in m \text{ or } n = m \text{ or } m \in n.$

- (6) Let $n \in \mathbb{N}$ and let R_0 and R_1 be two total orderings of the set n. Show that $(n, R_0) \cong (n, R_1)$ (i.e., there is an isomorphism between (n, R_0) and (n, R_1)).
- (7) A set I is called Zermelo-inductive if $\emptyset \in I$ and if $x \in I$, then $\{x\} \in I$. Show that if there is a Zermelo-inductive set, then there is a least Zermelo-inductive set (i.e., a Zermelo-inductive set M that is a subset of all Zermelo-inductive sets). Show that this set is transitive, but has non-transitive elements. Formulate an Induction Theorem for this set (in analogy to the Theorem of Complete Induction on \mathbb{N}).