Homework Sheet #1

MasterMath: Set Theory 2020/21: 1st Semester K. P. Hart, Benedikt Löwe, Ezra Schoen, & Ned Wontner

Deadline for Homework Set #1: Monday, 14 September 2020, 2pm. Please hand in via the elo webpage as a single pdf file.

- (1) Consider the natural numbers \mathbb{N} as a set of vertices in a graph with the edge relation E defined by n E m if and only if n < m. Check whether the axioms of extensionality, pairing, union, power set and the axiom scheme of separation hold in the structure (\mathbb{N}, E) .
- (2) Let $\mathbf{G} = (V, E)$ be the following graph model:



Check whether the axioms of extensionality, pairing, union, power set and the axiom scheme of separation hold in G.

- (3) Find a finite directed graph $\mathbf{G} = (V, E)$ that satisfies the axioms of extensionality, pairing, power set, and union. By a theorem from class, it cannot satisfy the axiom scheme of separation. Give a concrete instance of separation that fails in your graph.
- (4) Consider the following axiom of binary unions:

$$\forall x \forall y \exists u \forall z (z \in u \leftrightarrow (z \in x \lor z \in y)).$$

Show that every graph that satisfies the axioms of pairing and union also satisfies the axiom of binary union.