Homework Set #2

MasterMath: Set Theory
2019/20: 1st Semester
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Deadline for Homework Set #2: Monday, 23 September 2019, 2pm.

(5) We constructed a locally finite model $\mathcal{G}_\infty = (V_\infty, E_\infty)$ of FST in class. In class, we only sketched the argument why this is a model of the Separation Axiom Scheme. Give the argument in full detail.

(6) A set I is called *Zermelo-inductive* if $\varnothing \in I$ and if $x \in I$, then $\{x\} \in I$. Show that if there is a Zermelo-inductive set, then there is a least Zermelo-inductive set (i.e., a Zermelo-inductive set M that is a subset of all Zermelo-inductive sets). Show that this set is transitive, but has non-transitive elements.

(7) A vertex v in a directed graph is called *selflooping* if $v \; E \; v$. Suppose that $\mathcal{G} = (V, E) \models \mathsf{Z}$ and that $i \in V$ is inductive and furthermore that $s \in V$ is a vertex all of whose predecessors are selflooping. Show that $i \cup s$ is inductive.

(8) Prove that for any two natural numbers n and m one of the following holds:

$$n \in m \text{ or } n = m \text{ or } m \in n.$$

(9) We have defined the structure $(\mathbb{N}, +, \cdot, 0)$ in the theory Z. Give set-theoretic constructions (using only the axioms of Z) of the standard structures $(\mathbb{Z}, +, \cdot, 0)$ and $(\mathbb{Q}, +, \cdot, 0)$.

(*Hint.* Use the standard constructions for adding inverses to a structure from algebra and check that these constructions can be done in Z.)