Homework Set #1

MasterMath: Set Theory
2019/20: 1st Semester
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Homework should be handed in before the start of class on Monday (2pm). If handing in by e-mail, please submit homework to `e.j.h.wontner@uva.nl`. Each student has to write their own solutions entirely independently. Each attempted homework question will count 1 mark, independent of whether the solution is correct or not. The homework grade will be

$$\frac{\text{total number of attempted questions}}{\text{total number of questions}} \times 10$$

and counts 10% towards the final grade.

Deadline for Homework Set #1: Monday, 16 September 2019, 2pm.

(1) Consider a directed graph $\mathcal{G} = (V, E)$ and define the relation

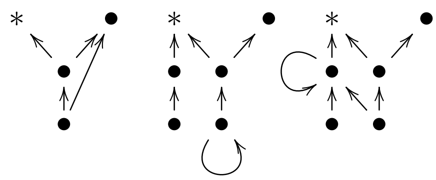
$$v \simeq w :\iff \forall z (z \; E \; v \leftrightarrow z \; E \; w).$$

Show that $\simeq$ is an equivalence relation and let V^* be the set of $\simeq$-equivalence classes. Define the following binary relation E^* on V^*:

$$C \; E^* \; D :\iff \exists v \in C \exists w \in D (v \; E \; w).$$

Does (V^*, E^*) satisfy the Axiom of Extensionality in general? If not, what goes wrong? If V is finite, argue that you can eventually obtain a graph that satisfies the Axiom of Extensionality by iterating this process.

(2) Consider the following graphs $\mathcal{G}_0$, $\mathcal{G}_1$ and $\mathcal{G}_2$ and determine the $\mathcal{G}_i$-subsets of the vertex marked by $*$ (for $i = 0, 1, 2$).



$\mathcal{G}_0$ $\mathcal{G}_1$ $\mathcal{G}_2$

(3) Consider the natural numbers $\mathbb{N}$ as a set of vertices in a graph with the edge relation E defined by $n \, E \, m$ if and only if $n < m$. Check whether the axioms Ext, Pair, Un, Pow, and Sep are valid in the structure $(\mathbb{N}, E)$.

(4) Assume that $\mathcal{G} = (V, E)$ is a directed graph that satisfies the Axiom Scheme of Separation. Let $x \in V$; by a result from class, x cannot be universal, so there must be a vertex that is not an element of x. Give a concrete definition of this vertex. (Hint. Use Russell's paradox.)