(5) Give a formula of $\mathcal{L}_\in$ (please do not use any abbreviations, only the logical symbols and = and $\in$) that describes the concept “$x$ has at least five elements that are sets of non-empty sets” in models of finite set theory.

(6) An element of a model of finite set theory $x$ is called irregular if $x \in x$. Suppose that $I$ is an inductive set and that $Z$ is a set of irregular elements. Show that $I \cup Z$ is inductive.

(7) A set $I$ is called Zermelo-inductive if $\emptyset \in I$ and if $x \in I$, then $\{x\} \in I$. Show that if there is a Zermelo-inductive set, then there is a least Zermelo-inductive set (i.e., a Zermelo-inductive set $M$ that is a subset of all Zermelo-inductive sets). Show that this set is transitive, but has non-transitive elements.

(8) We defined addition and multiplication on $\mathbb{N}$ by recursion as follows:

\[
\begin{align*}
  n + 0 & := n, \\
  n + S(m) & := S(n + m); \\
  n \cdot 0 & := 0, \\
  n \cdot S(m) & := (n \cdot m) + n.
\end{align*}
\]

Prove that both addition and multiplication are commutative operations.

(9) We defined $\mathbb{Z} := \{[(n, m)] \sim ; n, m \in \mathbb{N}\}$ where $(n, m) \sim (n', m')$ if and only if $n + m' = m + n'$. Define operations of addition and multiplication on $\mathbb{Z}$ that behave like they should and prove that your operations are well-defined and commutative.