Homework Set #2

MasterMath: Set Theory
2018/19: 1st Semester
K. P. Hart, Benedikt Löwe, & Robert Paßmann

Deadline for Homework Set #2: Monday, 24 September 2018, 2pm.

(5) Give a formula of $\mathcal{L}_\in$ (please do not use any abbreviations, only the logical symbols and $=$ and $\in$) that describes the concept "x has at least five elements that are sets of non-empty sets" in models of finite set theory.

(6) An element of a model of finite set theory x is called *irregular* if $x \in x$. Suppose that I is an inductive set and that Z is a set of irregular elements. Show that $I \cup Z$ is inductive.

(7) A set I is called *Zermelo-inductive* if $\varnothing \in I$ and if $x \in I$, then $\{x\} \in I$. Show that if there is a Zermelo-inductive set, then there is a least Zermelo-inductive set (i.e., a Zermelo-inductive set M that is a subset of all Zermelo-inductive sets). Show that this set is transitive, but has non-transitive elements.

(8) We defined addition and multiplication on $\mathbb{N}$ by recursion as follows:

$$\begin{aligned} n + 0 &:= n, \\ n + S(m) &:= S(n+m); \end{aligned}$$

$$\begin{aligned} n \cdot 0 &:= 0, \\ n \cdot S(m) &:= (n \cdot m) + n. \end{aligned}$$

Prove that both addition and multiplication are commutative operations.

(9) We defined

$$\mathbb{Z} := \{[(n,m)]_\sim\,;\, n,m \in \mathbb{N}\}$$

where $(n,m) \sim (n',m')$ if and only if $n + m' = m + n'$. Define operations of addition and multiplication on $\mathbb{Z}$ that behave like they should and prove that your operations are well-defined and commutative.