Homework Set #13

MasterMath: Set Theory
2018/19: 1st Semester
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Deadline for Homework Set #13: Monday 10 December 2018, 2pm.

(54) Let X be a Polish space, with topology τ, and F a closed subset of X. Let σ be the smallest topology that contains $\tau \cup \{F\}$. Prove that σ is a Polish topology and that the Borel sets defined from σ are exactly the Borel sets defined from τ.

(55) Let $\{A_n : n \in \omega\}$ be a pairwise disjoint family of analytic subsets of X. Prove that there is a pairwise disjoint family $\{C_n : n \in \omega\}$ of Borel sets such that $A_n \subseteq C_n$ for all n.

(56) Let $\{A_s : s \in \mathrm{Seq}\}$ be a family of Borel sets such that $A_s \supseteq A_t$ whenever $s \subseteq t$ and such that $A_{s^\frown n} \cap A_{s^\frown m} = \emptyset$, whenever $s \in \mathrm{Seq}$ and $n \neq m$. Prove that $\bigcup_{a \in \mathcal{N}} \bigcap_{n \in \omega} A_{a \restriction n}$ is a Borel set. *Hint*: Try to interchance $\bigcup$ and $\bigcap$.

(57) Prove that there are a set of measure zero Z and a meager set M such that $\mathbb{R} = Z \cup M$. *Hint*: Inspect the proof in class that $\mu^*(\mathbb{Q}) = 0$.

(58) Let $r \in (0, 1)$. Construct a closed nowhere dense subset D_r of $[0, 1]$ such that $\mu(D_r) = r$.