Homework Set #12

MasterMath: Set Theory
2018/19: 1st Semester
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Deadline for Homework Set #12: Monday 03 December 2018, 2pm.

(51) Let Z be the subset of $\mathcal{N}$ defined by

$$z \in Z \text{ if and only if } \lim_{n\to\infty} z(n) \cdot 2^{-n} = 0$$

Show that Z is a Borel subset of $\mathcal{N}$. *Hint*: Z is in fact a Π^0_3-set.

(52) The number of Borel and analytic sets.
- a. Prove that the family of open sets in $\mathbb{R}$ has cardinality $2^{\aleph_0}$.
- b. Prove that for every $\alpha < \omega_1$ the family of Σ^0_α-sets in $\mathbb{R}$ has cardinality $2^{\aleph_0}$.
- c. Prove that the σ-algebra of Borel sets in $\mathbb{R}$ has cardinality $2^{\aleph_0}$.
- d. Prove that the family of analytic subsets of $\mathbb{R}$ ha scardinality $2^{\aleph_0}$.

(53) Other universal sets.
- a. Let $\alpha < \omega_1$. Prove: if $U \subseteq \mathcal{N}^2$ is a universal Σ^0_α-set then $\mathcal{N}^2 \setminus U$ is a universal Π^0_α-set.
- b. Let $F \subseteq \mathcal{N}^3$ be a universal closed set, that is: F is closed and for every closed subset G of $\mathcal{N}^2$ there is a $z \in \mathcal{N}$ such that $G = \{\langle x, y\rangle : \langle x, y, z\rangle \in F\}$. Let $A = \{\langle x, z\rangle : (\exists y)(\langle x, y, z\rangle \in F)\}$. Prove that A is a universal analytic set.
- c. Prove that there is an analytic subset of $\mathcal{N}$ whose complement is not analytic.