Homework Set #1

MasterMath: Set Theory
2018/19: 1st Semester
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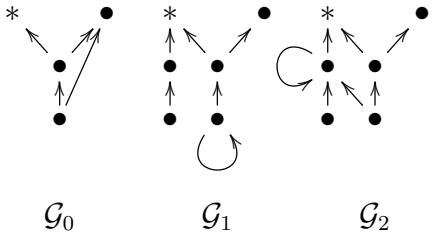
Homework should be handed in before the start of class on Monday (2pm). If handing in by e-mail, please submit homework to `r.passmann@uva.nl`. Each student has to write their own solutions entirely independently. Each attempted homework question will count 1 mark, independent of whether the solution is correct or not. The homework grade will be

$$\frac{\text{total number of attempted questions}}{\text{total number of questions}} \times 10$$

and counts 10% towards the final grade.

Deadline for Homework Set #1: Monday, 17 September 2018, 2pm.

(1) Consider the following graphs $\mathcal{G}_0$, $\mathcal{G}_1$ and $\mathcal{G}_2$ and determine the $\mathcal{G}_i$-subsets of the vertex marked by $*$ (for $i = 0, 1, 2$).



(2) Consider the natural numbers $\mathbb{N}$ as a set of vertices in a graph with the edge relation E defined by $n\,E\,m$ if and only if $n \leq m$. Check the validity of the axioms Ext, Pair, Un, Pow, and Sep in the structure $(\mathbb{N}, E)$.

(3) Find a finite directed graph $\mathcal{G} = (V, E)$ that satisfies Ext, Pair, Pow, and Un. By a theorem from class, it cannot satisfy Sep. Give a concrete instance of Sep that fails in your graph.

(4) Consider the following *Axiom of binary unions* BinUn:

$$\forall x \forall y \exists u \forall z (z \in u \leftrightarrow (z \in x \vee z \in y)).$$

Show that every graph that satisfies both Pair and Un also satisfies BinUn.