HOMEWORK 12

SET THEORY

▶ **1** (Jech: 9.4). Prove that for all infinite cardinals κ we have $\kappa \not\to (\omega)^{\omega}_{2}$.

Hint (slightly different from the book: define an equivalence relation on $[\kappa]^{\omega}$ by $s \equiv t$ iff the symmetric difference, $(s \setminus t) \cup (t \setminus s)$, is finite. Choose a set of representatives for the equivalence classes. Define a colouring F by $F(s) = 1$ if s and its representative differ by an even number of elements, $F(s) = 0$ otherwise. Now prove there is no infinite set H such that F is constant on $[H]^{\omega}$.

Note: this shows why in partition theory one generally looks at finite exponents only.

▶ **2** (An application of Theorem 9.6). In a partially ordered set, $(P, \le)$ we say that two elements, p and q, are *disjoint* if there is no $x \in P$ such that $x \le p$ and $x \le q$. An *antichain* in P is a set of pairwise disjoint elements.

If $(P, \le_P)$ and $(Q, \le_Q)$ are two partial orders then we order the product $P \times Q$ by $(p, q) \le (x, y)$ iff $p \le_P x$ and $q \le_Q y$.

Prove: if all antichains in P and in Q are countable then all antichains in $P \times Q$ have cardinality at most $2^{\aleph_0}$.

▶ **3** (Application of Theorem 9.7). Let κ be an infinite cardinal and let $\prec$ be some well-order of κ. Prove that there is a subset H of κ of cardinality κ such that $\prec$ agreees with the normal well-order of κ. (This means: for all $x, y \in H$ we have $x < y$ iff $x \prec y$.)

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