HOMEWORK 11

SET THEORY

▶ **1** (Jech: 9.1). Prove the following

(a) Every infinite partially ordered set has an infinite chain or an infinite set of mutually incomparable elements.

(b) Every infinite linearly ordered set has an infinite increasing sequence or an infinite decreasing sequence

▶ **2** (Jech 9.3). Prove that $\omega_1 \to (\omega_1, \omega+1)^2$.

Note: the task is to show that for every partition $\{A, B\}$ of $[\omega_1]^2$ there is either an uncountable subset H of ω_1 such that $[H]^2 \subseteq A$ or a subset K of ω_1 of order-type $\omega+1$ such that $[K]^2 \subseteq B$.

Hint, adapted from the book: show that one can choose for every α a maximal subset K_α of α such that $[K_\alpha \cup \{\alpha\}]^2 \subseteq B$. If some K_α is infinite then we are done; if every K_α is finite then use the Pressing-Down Lemma to find a stationary set and one finite set K such that $K_\alpha = K$ for all $\alpha \in S$. Show that $[S]^2 \subseteq A$.

▶ **3** (Special case of Lemma 9.3). Prove $2^{\aleph_0} \nrightarrow (3)^2_{\aleph_0}$ using the structure of $\mathbb{R}$. *Hint*: Let $\langle q_n : n < \omega \rangle$ be an enumeration of $\mathbb{Q}$ and define $F : [\mathbb{R}]^2 \to \omega$ by $F(\{x, y\}) = \min\{n : q_n$ lies between x and $y\}$.

Date: 2017-11-13.