HOMEWORK 10

SET THEORY

► **1** (Jech: 5.18, 5.24, 5.25). Prove the following

(a) $\aleph_\omega^{\aleph_1} = \aleph_\omega^{\aleph_0} \cdot 2^{\aleph_1}$

(b) If $2^{\aleph_0} > \aleph_\omega$ then $\aleph_\omega^{\aleph_0} = 2^{\aleph_0}$.

(c) If $2^{\aleph_1} = \aleph_2$ and $\aleph_\omega^{\aleph_0} > \aleph_{\omega_1}$ then $\aleph_{\omega_1}^{\aleph_1} = \aleph_\omega^{\aleph_0}$.

► **2** (Jech 8.2 and more). Let κ be regular and uncountable. Prove:

(a) If $f : \kappa \to \kappa$ is a function then $\{\alpha < \kappa : f[\alpha] \subseteq \alpha\}$ is closed and unbounded.

(b) If $n < \omega$ and $f : \kappa^n \to \kappa$ is a function then $\{\alpha < \kappa : f[\alpha^n] \subseteq \alpha\}$ is closed and unbounded.

(c) Let $\langle f_i : i < \omega\rangle$ be a sequence of functions, where $f_i : \kappa^{n_i} \to \kappa$ for some $n_i < \omega$. The set $\{\alpha : (\forall i)(f_i[\alpha^{n_i}] \subseteq \alpha\}$ is closed and unbounded.

(d) Let $*$ be a group operation on ω_1. The set $\{\alpha : \langle \alpha, *\rangle$ is a subgroup of $\langle \omega_1, *\rangle\}$ is closed and unbounded.

► **3** (M. E. Rudin: an elementary construction of two disjoint stationary sets in ω_1). Let $f : \omega_1 \to \mathbb{R}$ be an injective map. For $q \in \mathbb{Q}$ put $A_q = \{\alpha : f(\alpha) < q\}$ and $B_q = \{\alpha : f(\alpha) > q\}$. Let $I = \{q : A_q$ contains a cub set$\}$ and $J = \{q : B_q$ contains a cub set$\}$.

(a) Prove: if $p \in I$ and $q \in J$ then $q < p$.

(b) Prove: $I \neq \mathbb{Q}$ and $J \neq \mathbb{Q}$.

(c) Prove: $\sup J < \inf I$ (by convention: $\sup \emptyset = -\infty$ and $\inf \emptyset = \infty$).

(d) Prove: there is a $q \in \mathbb{Q}$ such that both A_q and B_q are stationary.

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