# HOMEWORK 9

SET THEORY

▶ **1** (Jech: 5.11, 5.12, 5.13, and 5.14). Prove the following equalities:

(a) $\prod_{0<n<\omega} n = 2^{\aleph_0}$;

(b) $\prod_{n<\omega} \aleph_n = \aleph_\omega^{\aleph_0}$;

(c) $\prod_{\alpha<\omega+\omega} \aleph_\alpha = \aleph_{\omega+\omega}^{\aleph_0}$;

(d) If $\mathsf{GCH}$ holds then

- $2^{<\kappa} = \kappa$ for all $\kappa$, and
- $\kappa^{<\kappa} = \kappa$ for all *regular* $\kappa$

▶ **2** (Jech 5.15). If $\beta$ is such that $2^{\aleph_\alpha} = \aleph_{\alpha+\beta}$ for every $\alpha$, then $\beta < \omega$.

▶ **3** (Jech 5.17). Show that if $\kappa$ is a limit cardinal and $\lambda < \operatorname{cf} \kappa$ then $\kappa^\lambda = \sum_{\alpha<\kappa} |\alpha|^\lambda$.

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