HOMEWORK 2

SET THEORY

► **1** (Jech: 1.14). Prove: The Separation Axioms follow from the Replacement Axioms.

► **2** (Jech: 1.15). Use the Separation Schema to prove the Union Axiom, the Power Set Axiom and the Replacement Axioms from their weaker versions:

(1) $(\forall X)(\exists Y)(\bigcup X \subseteq Y)$ or rather $(\forall X)(\exists Y)(\forall x \in X)(\forall u)(u \in x \to u \in Y)$
(2) $(\forall X)(\exists Y)(\mathcal{P}(X) \subseteq Y)$ or rather $(\forall X)(\exists Y)(\forall u)(u \subseteq X \to u \in Y)$
(3) if a class F is a function then $(\forall X)(\exists Y)(F[X] \subseteq Y)$

► **3**. The bottom of page 13 contains our definitions of $\mathbb{N}$, the order $<$, and of $n+1$ as $n \cup \{n\}$. It also defines the notion of a *transitive set*: T is transitive if every element of T is a subset of T, that is if $x \in T$ and $y \in x$ then $y \in T$.

Prove the following about inductive sets: if X is inductive then so are

(1) $\{x \in X : x \subseteq X\}$
(2) $\{x \in X : x \text{ is transitive}\}$
(3) $\{x \in X : x \text{ is transitive and } x \notin x\}$
(4) $\{x \in X : x$ is transitive and every nonenmpty subset of x has an $\in$-minimal element$\}$
(5) $\{x \in X : x = \emptyset \text{ or } x = y \cup \{y\} \text{ for some } y\}$

► **4**. We denote by A^n the set of functions from the natural number n to the set A. We have seen that this set can be proven to exist on the basis of the axioms up to and including the Power Set axiom. In this exercise we drop the Power Set axiom, but adopt the Axiom of Infinity and the Replacement Schema. Prove that the following sets exist, given a fixed set A:

(1) the set A^n, for each $n \in \mathbb{N}$
(2) the function $n \mapsto A^n$
(3) the set $\bigcup\{A^n : n \in \mathbb{N}\}$ of all finite sequences of members of A
(4) the set $[A]^n$ of subsets of A with n elements, as defined on page 14 of the book
(5) the set $[A]^{<\omega}$ of all finite subsets of A, (with ‘finite’ as defined on page 14 of the book)

If you submit by email, use this address: `lorenzo.galeotti@gmail.com`

Date: 2017-09-18.