

HOMEWORK 7

SET THEORY

- ▶ **1** (Jech: 4.2). Show that there are at least \mathfrak{c} many countable order-types of linearly ordered sets.
- ▶ **2** (Jech: 4.14). Show that \mathbb{Q} is not the intersection of countably many open sets.
- ▶ **3**. Given a set X of real numbers we define by transfinite recursion the following sequence:

$$X_0 = X,$$

$$X_{\alpha+1} = X'_\alpha,$$

$$X_\lambda = \bigcap_{\alpha < \lambda} X_\alpha \text{ for } \lambda \text{ limit.}$$

We call *Cantor-Bendixson rank* of X the smallest ordinal α such that $X_\alpha = X_{\alpha+1}$. Give examples of sets X such that

- (a) X has Cantor-Bendixson rank 2;
- (b) X has Cantor-Bendixson rank 3.