Homework Set #1

Axiomatische Verzamelingentheorie
2012/13: 2nd Semester
Universiteit van Amsterdam

The homework can be handed it electronically to Takanori Hida (`t.hida@uva.nl`) or on paper in the Friday lectures. Please hand in the homework before the start of the Friday lecture (1pm). Late homework will not be accepted.

The homework should contain the full names and student ID numbers of all students who contributed. Each homework solution should have at most two names of students. The homework handed in must be the work of the students named on that homework solution. If your homework is handwritten, make sure that it is legible. Also, make sure that you write in complete English sentences.

This homework set is due on **Friday 15 February 2013, 1pm**.

1. Remember the balls-and-box paradox from the first lecture: "You have infinitely many balls and a box. You start by putting ten balls into the box. After that, in each step, you first remove a ball and then put ten new balls in. How many balls are in the box after infinitely many steps?"

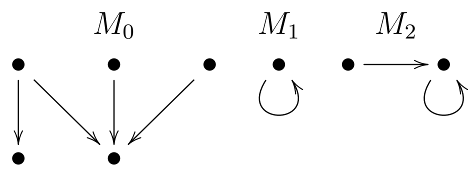
   In the lecture, we saw that depending on the mathematical representation, the answer can be "infinitely many" or "none". Show that for any natural number n, the answer can be n.

   (**Hint.** If $n < 10$, the solution is very easy. If $n \geq 10$, then some more work is required.)

2. In the lecture, we said that *models* are just directed graphs where the edge relation of the graph is interpreted as "is an element of". Show that there is a model of (Ex), (Ext), (Aus) together with the statement $\exists x(\varnothing \in x)$.

   (**Hint.** In order to show that this is a model, you need to give a precise definition of the model, e.g., by drawing the graph with appropriate labels for the vertices, and prove the validity of each of the four statements.)

3. Consider the following three models:



   For each of them, check whether (Ext), (Aus), $\exists x \forall y(y \in x)$, and $\exists x \forall y(y \notin x)$ holds in these models.