UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

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Homework Set # 3 *Deadline:* October 3rd, 2007

Exercise 9 (9 points).

We are returning once more to the world of sheep and owners from Exercise 7. The following are two possible formalizations of the 'birth cycle' of Exercise 7:

Formalization 1. Let P, S be finite pairwise disjoint sets (representing 'people', 'sheep') Let $B \subseteq S$ be the set of 'black sheep' and $W \subseteq S$ be the set of 'white sheep'. Let $O \subseteq S \times P$ be the ownership relation. We define the semantics for our syntactic objects as follows: $\mathbf{black}(s)$ if and only $s \in B$; $\mathbf{white}(s)$ if and only if $s \in W$; $\mathbf{owner}(p, s)$ if and only if $\langle s, p\rangle \in O$.
We call $\langle P, S, B, W, O\rangle$ an **ovine model** if $W \cap B = \varnothing$ and $W \cup B = S$ and if for all $p, q \in P$ and $s \in S$, we have that if $\langle s, p\rangle \in O$ and $\langle s, q\rangle \in O$, then $p = q$.
Suppose that $S = S_0 \cup S_1$ where $S_0 \cap S_1 = \varnothing$ (representing 'old sheep' and 'new sheep'). A relation $R \subseteq S_0 \times S_1$ is called a **birth relation** if it has the following properties:

- If $s \in S_0 \cap B$ and $\langle s, t\rangle \in R$, then $t \in S_1 \cap B$.
- If $s \in S_0 \cap W$ and $\langle s, t\rangle \in R$, then $t \in S_1 \cap W$.
- If $\langle s, p\rangle \in O$ and $\langle s, t\rangle \in R$, then $\langle t, p\rangle \in O$.
- For every $t \in S_1$ there is exactly one $s \in S_0$ such that $\langle s, t\rangle \in R$.

We say that an ovine model is an **implicit birth cycle model** if there is a partition $S = S_0 \cup S_1$ such that S_0 has exactly twice as many elements as S_1 and there is a birth relation $R \subseteq S_0 \times S_1$. We interpret $\langle s, t\rangle \in R$ as "s gives birth to t".

Formalization 2. Let P, $S = S_0 \cup S_1$, B, W, O as in Formalization 1. We call a model $\langle P, S_0, S_1, R, B, W, O\rangle$ an **explicit birth cycle model** if

- $\langle P, S_0 \cup S_1, B, W, O\rangle$ is an ovine model,
- R is a birth relation,
- S_0 has exactly twice as many elements as S_1.

Describe briefly the difference between Formalization 1 and Formalization 2. Pay particular attention to the question whether a relation "is the mother of" can be defined in implicit or explicit birth cycle models. (2 points)

Now suppose that you are told that "10% of all sheep die of old age" after the birth cycle. Which of the two formalizations would you choose as the starting point to formalize this statement and why? (1 point) Would this change if the additional information was "10% of all *old* sheep die of old age"? Discuss. (1 point)

Give a precise definition of a class of models that you call **birth and old age model** and that capture "10% of all sheep die of old age". (1½ points) Give an example of a birth and old age model in which the conclusion of Exercise 7 is violated, i.e., no shepherd owns only white sheep at the beginning, but there is a shepherd who owns only white sheep after the birth and old age steps. (1 point)

Now suppose that instead of old age, you have the additional information "25% of all sheep giving birth die". Which of the two formalizations would you now choose as a the starting point to formalize this statement and why? (1 point) Give a precise definition of a class of models that you call **birth and death model** and that capture this information. (1½ points)

Exercise 10 (7 points).

The following is a syntax for a term logic in the Aristotelean style. We have term variables $\mathtt{t}_i$ and symbols $\mathbf{All}$, $\mathbf{Some}$, $\mathbf{No}$ and $\mathbf{Somenot}$, where, *e.g.*, $\mathbf{Some}(\mathtt{t}_0, \mathtt{t}_1)$ is interpreted as "Some $\mathtt{t}_0$ is $\mathtt{t}_1$," *etc.* If $\mathbf{S}$ is one of the four symbols and $\mathtt{t}_0$ and $\mathtt{t}_1$ are term variables, then $\mathbf{S}(\mathtt{t}_0, \mathtt{t}_1)$ is called a *clause*. We now add two operators $+$ and $-$ to the language. If C is a clause, then both $+C$ and $-C$ are *statements*, interpreted as "C is true" and "C is false", respectively.

A *rule* for this syntax is a diagram

$$\frac{S}{S'}$$

where S and S' are statements. For instance,

$$\frac{+\mathbf{All}(\mathtt{t}_0, \mathtt{t}_1)}{-\mathbf{No}(\mathtt{t}_0, \mathtt{t}_1)}$$

is a rule, interpreted as "if all $\mathtt{t}_0$ are $\mathtt{t}_1$, then it cannot be true that no $\mathtt{t}_0$ is $\mathtt{t}_1$".

Give all of the rules corresponding to the relationships represented in the square of opposition (for example, the above rule corresponds to one instance of "contraries cannot both be true at the same time").

Exercise 11 (6 points; $+2$ extra points)

Lacydes of Cyrene was one of the heads of the Academy. The *Encyclopædia Britannica* reports:

> According to Athenaeus (x. 438) and Diogenes Laertius (iv. 60) he died from excessive drinking, but the story is discredited by the eulogy of Eusebius (Praep. Ev. xiv. 7), that he was in all things moderate.

Find the original quote of Diogenes Laertius (3 points; if you quote it in the original language, this is worth 2 extra points). Find the "eulogy of Eusebius". Are you convinced that it discredits the story reported by Athenaeus and Diogenes Laertius? Give an argument for your position. (3 points)