

Aristotelian modal logic.

Modalities.

- $\mathbf{A}p \simeq$ “ p ” (no modality, “assertoric”).
- $\mathbf{N}p \simeq$ “necessarily p ”.
- $\mathbf{P}p \simeq$ “possibly p ” (equivalently, “not necessarily not p ”).
- $\mathbf{C}p \simeq$ “contingently p ” (equivalently, “not necessarily not p and not necessarily not p ”).

Every (assertoric) mood $p, q : r$ represents a modal mood $\mathbf{A}p, \mathbf{A}q : \mathbf{A}r$. For each mood, we combinatorially have $4^3 = 64$ modalizations, i.e., $256 \times 64 = 16384$ modal moods.

Modal conversions.

● Simple.

- $NXeY \rightsquigarrow NYeX$
- $NXiY \rightsquigarrow NYiX$
- $CXeY \rightsquigarrow CYeX$
- $CXiY \rightsquigarrow CYiX$
- $PXeY \rightsquigarrow PYeX$
- $PXiY \rightsquigarrow PYiX$

● Accidental.

- $NXaY \rightsquigarrow NXiY$
- $CXaY \rightsquigarrow CXiY$
- $PXaY \rightsquigarrow PXiY$
- $NXeY \rightsquigarrow NXoY$
- $CXeY \rightsquigarrow CXoY$
- $PXeY \rightsquigarrow PXoY$

● Relating to the symmetric nature of contingency.

- $CXiY \rightsquigarrow CXeY$
- $CXeY \rightsquigarrow CXiY$
- $CXaY \rightsquigarrow CXoY$
- $CXoY \rightsquigarrow CXaY$

● $NXxY \rightsquigarrow AXxY$ (Axiom T: $\Box\varphi \rightarrow \varphi$)

Modal axioms.

What are the “perfect modal syllogisms”?

- Valid assertoric syllogisms remain valid if **N** is added to all three propositions.

Barbara ($AaB, BaC:AaC$) \rightsquigarrow **NNN Barbara** ($NAaB, NBaC:NAaC$).

First complications in the arguments for **Bocardo** and **Baroco**.

- By our conversion rules, the following can be added to valid assertoric syllogisms:
 - **NNA**,
 - **NAA**,
 - **ANA**.
- Anything else is problematic.

The “two Barbaras”.

NAN Barbara

$NAaB$

$ABaC$

$NAaC$

ANN Barbara

$AAaB$

$NBaC$

$NAaC$

From the modern point of view, both modal syllogisms are invalid, yet Aristotle claims that **NAN Barbara** is valid, but **ANN Barbara** is not.

De dicto versus De re.

We interpreted $\mathbf{N}AaB$ as

“The statement ‘ AaB ’ is necessarily true.”

(*De dicto* interpretation of necessity.)

Alternatively, we could interpret $\mathbf{N}AaB$ *de re* (Becker 1933):

“Every B happens to be something which is necessarily an A .”

Necessarily, every white bird is white.

Every swan is a white bird.

Ergo: Every swan happens to be something that is necessarily white.

Aristotelian temporal logic: the sea battle

According to the square of oppositions, exactly one of “it is the case that p ” and “it is not the case that p ” is true.

Either “it is the case that there will be a sea battle tomorrow”
or “it is not the case that there will be a sea battle tomorrow”.

Problematic for existence of free will, and for Aristotelian metaphysics.

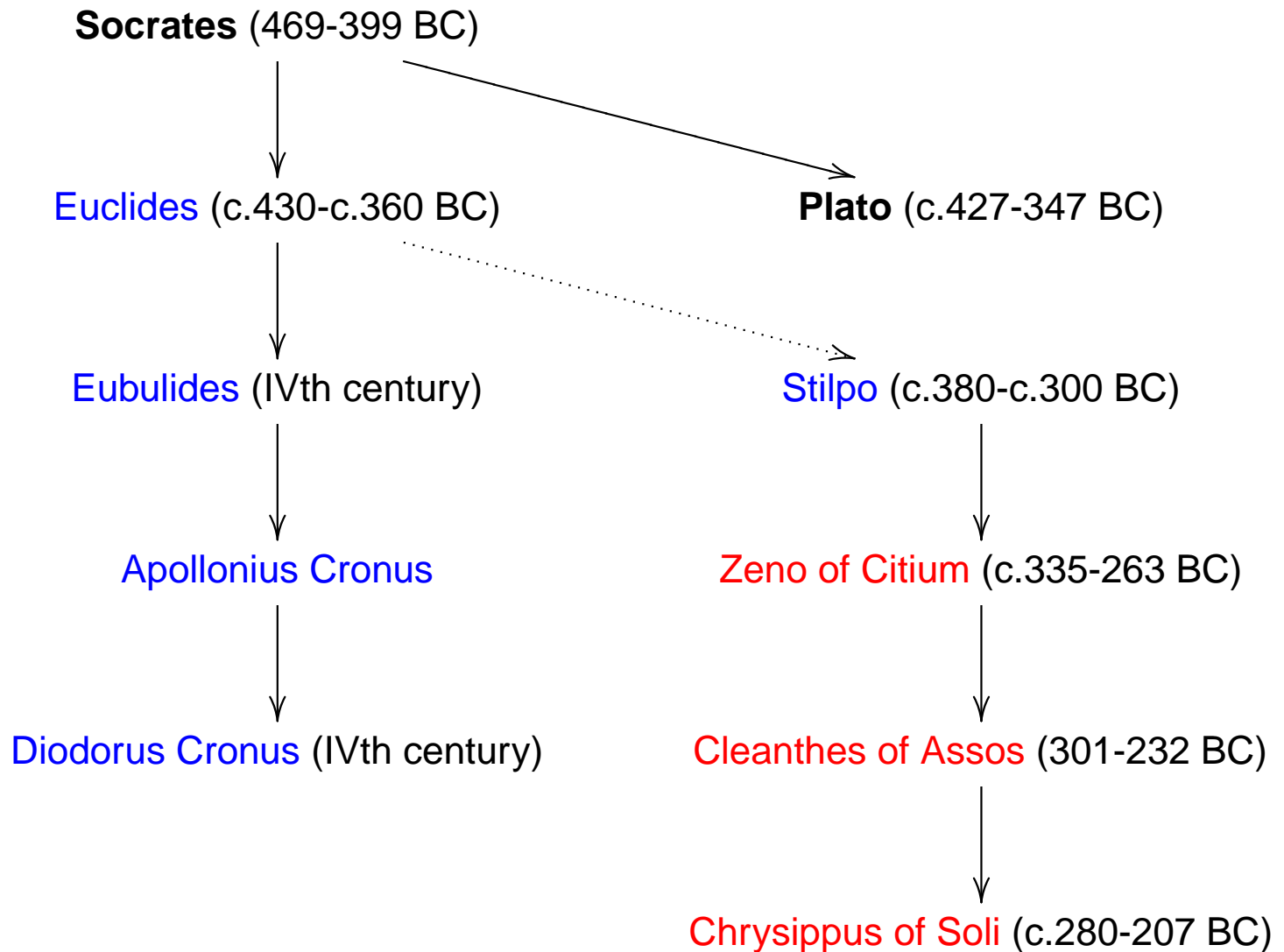
The Master argument.

Diodorus Cronus (IVth century BC).

- Assume that p is not the case.
- In the past, “It will be the case that p is not the case” was true.
- In the past, “It will be the case that p is not the case” was necessarily true.
- Therefore, in the past, “It will be the case that p ” was impossible.
- Therefore, p is not possible.

Ergo: Everything that is possible is true.

Megarians and Stoics.



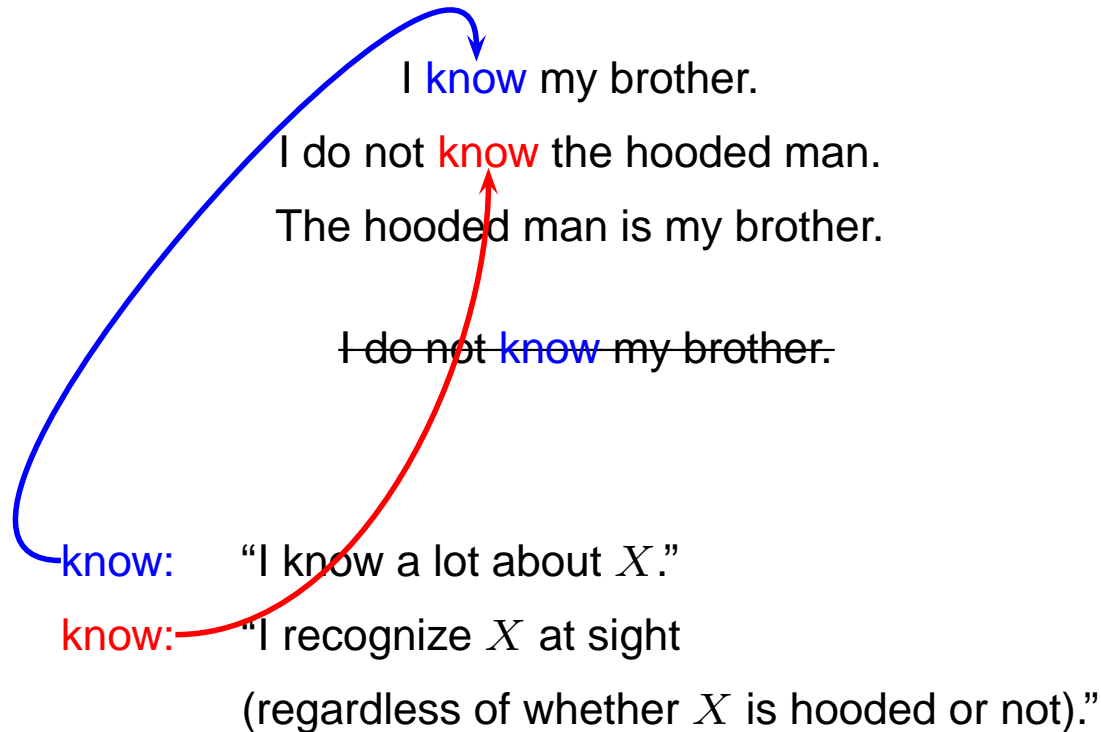
Eubulides.

- Source of the “seven Megarian paradoxes”, among them the *Liar*.
 - *The Liar* is attributed to Epimenides the Cretan (VIIth century BC); (*Titus 1:12*).
 - **Aulus Gellius**, *Noctes Atticae*.
Alessandro **Garcea**, Paradoxes in Aulus Gellius, **Argumentation** 17 (2003), p. 87-98
- Graham **Priest**, The Hooded Man, **Journal of Philosophical Logic** 31 (2002), p. 445-467

The seven Megarian paradoxes.

- *The Liar.* “Is the man a liar who says that he tells lies?”
- *The concealed man.* “Do you know this man who is concealed? If you do not, you do not know your own father; for he it is who is concealed.”
- *The hooded man.* “You say that you know your brother. Yet that man who just came in with his head covered is your brother and you did not know him.”
- *Electra.* “Electra sees Orestes : she knows that Orestes is her brother, but does not know that the man she sees is Orestes; therefore she does know, and does not know, her brother at the same time.”
- *The Sorites / the heap.* “One grain of wheat does not make a heap. Adding one grain of wheat doesn’t make a heap.”
- *The bald one.* “Pulling one hair out of a man’s head will not make him bald, nor two, nor three, and so on till every hair in his head is pulled out.”
- *The horned one.* You have what you have not lost. You have not lost horns, therefore you have horns.

Quarternio terminorum.



Every metal is a chemical element.

Brass is a metal.

~~Brass is a chemical element.~~

Shortcomings of syllogistics.

Syllogistics is finitary and cannot deal with very simple propositional connectives:

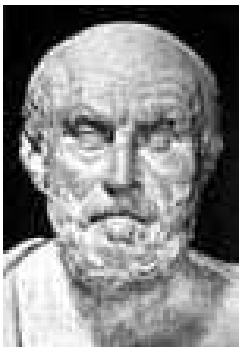
Every human being is a man or a woman.

Every man is mortal.

Every woman is mortal.

Ergo... every human being is mortal.

Stoic Logic.



Chrysippus of Soli (c.280-207 BC)

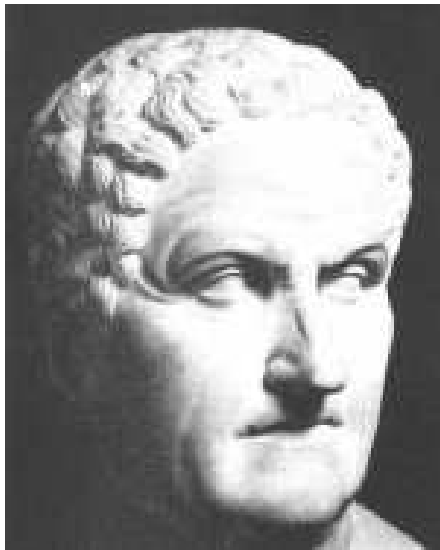
- 118 works on logic,
- seven books on *the Liar*,
- “inventor of propositional logic”,
- nonstandard view of modal logic (“the impossible can follow from the possible”).

Harry **Ide**, Chrysippus’s response to Diodorus’s master argument, **History and Philosophy of Logic** 13 (1992), p. 133-148.

Michael **Frede**, *Die stoische Logik*, Vandenhoeck & Ruprecht 1974

Late antiquity.

- Galen (129-216)



Galen of Pergamum

(129-216)

Court Physician to Marc Aurel

Introduction to Dialectics

(rediscovered in XIXth century)

Late antiquity.

- Galen (129-216)
- Augustine (354-430)



(Sanctus) Aurelius Augustinus
(354-430)
doctor ecclesiae

Late antiquity.

- Galen (129-216)
- Augustine (354-430)
- Boëthius (c.475-524)
- Cassiodorus (c.490-c.585)



Flavius Magnus Aurelius Cassiodorus Senator
(c.490-c.585)

Main work: *Institutiones*

Late antiquity.

- Galen (129-216)
- Augustine (354-430)
- Boëthius (c.475-524)
- Cassiodorus (c.490-c.585)
- Isidore of Seville (c.560-636)



(Sanctus) Isidorus Hispalensis
(c.560-636)

Main work: *Etymologiae*

Patron Saint of the Internet

Boëthius.



Anicius Manlius Severinus Boëthius
(c.475-524)

“The last of the Roman philosophers, and the first of the scholastic theologians” (Martin Grabmann)

Logic in the Middle Ages.

- *Peripatetic position:* Logic is a preliminary to scientific inquiry.
- *Stoic position:* Logic is part of philosophy.

In the Middle Ages:

- Logic as *ars sermocinalis*. (Part of the preliminary studies of the *trivium*.)
- Logic (in a broader sense) as central to important questions of philosophy, metaphysics and theology.

Theological Questions.

Theological questions connected with the set-up of logic.

- **The Existence of Evil.**
- **The Immortality of the Soul.**
- **The Eucharist.**
- **The Trinity and the ontological status of Father, Son and Holy Spirit.**
- **Free will and responsibility for one's actions.**

Platonic dualism (1).

- Matter-form (body-mind) dualism in Plato. (*Phaedo*.)
- Moral categories of *good* and *bad* mix with theological questions: “If God is good, and God created the world, how can there be Evil in the world?”
- *Augustine / Thomas Aquinas*. Evil is the absence of Good. Evil has no ontological status. (*privatio*).
- The opposite view: *Manicheism* (Mani of Persia; c.215-276). “Moral Dualism”: there are two principles at work; Good and Evil.

Platonic dualism (2).



(Sanctus) Aurelius Augustinus
(354-430)

doctor ecclesiae

Follower of Manicheism for nine years (375-384 AD) before rejecting these teachings under the influence of Ambrose.

Albigensians / Cathars.

- Southern France, XIth-XIIIth century.
- “Neomanicheism”, “Latin Manicheism”.
- Albigensian crusade (1209-1255); massacre of Béziers (1209): *Caedite eos. Novit enim Dominus qui sunt eius.* (Caesarius of Heisterbach c.1225)

The soul as form of the animal.

- A living being (plant, animal, human) is a compound of its matter and its form. The form is its capacity to engage in its characteristic activities (“first actuality” = instantiated skill): **Self-nourishment**, **Growth**, **Movement**, **Perception**, **Intellect**.
- The **nutritive soul**, the **sensitive soul**, and the **intellectual soul**.
- *De anima* (414a20): The soul does not exist without a body and yet is not itself a kind of body.

Two interpretations.

- “The individual form theory”. There are individual forms: the form (soul) of Socrates and the form (soul) of Aristotle. As soon as they are not instantiated anymore, they cease to exist.
- “Monopsychism”. There is one intellectual soul that is instantiated in all human beings. Individuality comes from being instantiated by matter.

Immortality of the Soul.

- *De anima* not known until early XIIIth century.
- Averroes (Ibn Rushd; 1126-1198).
- Averroism banned 1270 and 1277.
- *Radical Aristotelianism*: Siger of Brabant (d.1282), Boëthius of Dacia (d.1290), Dante Alighieri (1265-1321).
- Giovanni Pico della Mirandola (1463-1494).



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- Giovanni Pico della Mirandola (1463-1494).
- Giordano Bruno (1548-1600).

