UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2006/2007; 1st Semester
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Homework Set # 6 *Deadline:* October 18th, 2006

Exercise 18 (8 points).

Consider a set W of states and a set X of objects. We call the set $\widehat{X} := \{+,-\} \times X := \{\langle +, x\rangle \,;\, x \in X\} \cup \{\langle -, x\rangle \,;\, x \in X\}$ the set of **entities**. We think of $\langle -, x\rangle$ as the imagined object x and $\langle +, x\rangle$ as "$\langle -, x\rangle$ with the added property of existence". We call entities $\langle +, x\rangle$ **existing entities**.

For each $w \in W$, fix a set $X_w \subseteq \widehat{X}$ of **permissible entities** in w. We fix two strict linear ordering $<$ and $\prec$ on $\widehat{X}$, and an accessibility relation R on W. As in **Exercise 15**, we say that "v is conceivable from w" if wRv. For $x, y \in X_w$, we say "in w, x is better (bigger) than y" if $y < x$ ($y \prec x$). A structure $\mathbf{W} := \langle W, \langle X_w \,;\, w \in W\rangle, R, <, \prec\rangle$ is called **Anselmian** if it has the following properties:

- If wRv and $\widehat{x} \in X_w$, then $\widehat{x} \in X_v$.
- For each $w \in W$, if $\langle -, x\rangle \in X_w$, then there is some v such that wRv and $\langle +, x\rangle \in X_v$.
- For each $x \in X$, $\langle -, x\rangle < \langle +, x\rangle$.

If $\mathbf{W}$ is an Anselmian structure and $w \in W$, we say that an entity $\widehat{x} \in \widehat{X}$ is **Anselmian** in w if for all v such that wRv and all $\widehat{y} \in X_v$, it is not the case that $\widehat{x} < \widehat{y}$. We say that an entity $\widehat{x} \in \widehat{X}$ is **Gaunilan** in w if for all v such that wRv and all $\widehat{y} \in X_v$, it is not the case that $\widehat{x} \prec \widehat{y}$.

The second half of the ontological argument can now be rephrased as follows: In an Anselmian structure, every Anselmian entity is existing. Prove this statement. (1½ points)

Give an example of an Anselmian structure with a state w in which there is a nonexisting Gaunilan entity (*i.e.*, an entity of the form $\langle -, x\rangle$). (3 points)

There is a simple modification of the notion of an Anselmian structure that we could call a **Gaunilan structure**, for which we can prove that every Gaunilan entity is existing. Give a precise definition of this and prove the statement. (2 points)

Consider your definition of a Gaunilan structure. It is possible to justify the new axiom as "true" in some natural sense? Could you convince a nonbeliever of the axioms of your Gaunilan structure? Give a brief discussion (at most 10 lines; 1½ points).

Exercise 19 (6 points).

Consider the sentence *omnis philosophus praeter Socratem albus est* ("every philosopher except for Socrates is white".

Give a modern semantics for the *omnis praeter* construction: suppose we have a universe of discourse X, two predicates $\Phi, \Psi \subseteq X$ and $x \in X$. Give a formal definition such that

$$\mathbf{omnispraeter}(x, \Phi, \Psi)$$

is true if and only if *omnis* Φ *praeter* x *est* Ψ ("every Φ except for x is Ψ") (1 point).

Note. The "modern semantics" is not necessarily unique. There might be different semantics that describe the natural language sentences reasonably adequately.

Now consider the sophisma

$(\star)$ *omnis homo praeter Socratem excipitur*

("every man except for Socrates is excepted").

(1) Give a background story which describes a situation in which $(\star)$ is true (1 point).
(2) Argue informally that $(\star)$ is false (2 points).
(3) Solve the apparent contradiction by explaining the fallacy as a *secundum quid et simpliciter* (2 points).

Exercise 20 (3 points).

If X is any set and $\wp(X)$ is its power set (the set of all subsets of X), we call $\mathsf{Q} \subseteq \wp(X)$ a **generalized quantifier**. If $\Phi \subseteq X$ is a predicate on X, we say that $\mathsf{Q}\Phi$ holds (in words: "for Q-many x, $\Phi(x)$ holds") if $\Phi \in \mathsf{Q}$.

(1) Let $\forall := \{X\}$ and $\exists := \{A \subseteq X\,;\ A \neq \varnothing\}$. Argue that $\forall\Phi$ and $\exists\Phi$ have the intended meanings "for all x, $\Phi(x)$ holds" and "there is an x such that $\Phi(x)$ holds" (½ point each).
(2) Fix some $x \in X$ and give a definition of a generalized quantifier $\mathtt{op}_x$ that corresponds to the *omnis praeter* construction from **Exercise 19** (2 points).

Exercise 21 (5 points).

- Correct or false? (½ point each)
  (1) Giovanni Pico della Mirandola wrote the famous *oratio de hominis dignitate* which can be seen as a "manifesto of the Italian renaissance".
  (2) Before returning to Italy where he was going to be sentenced to death, Giordano Bruno spent some time in England.
  (3) Arius claimed that God-Father and God-Son have different substances, but both are eternal. This teaching was rejected in the Council of Nicaea in 325 AD.
  (4) Anselm of Canterbury and Lanfranc of Bec knew each other personally.
  (5) Johannes Scotus Eriugena wrote a book entitled *De gemina praedestinatione* on predestination in which he discusses the debate between Gottschalk and Hrabanus Maurus.
  (6) Despite their differences, Abelard speaks very highly of his former teacher Anselm of Laon in his *Historia Calamitatum Mearum*.
- Give the names of the following medieval logicians and philosophers (1 point each):
  – Y was one of the students of Anselm of Laon and taught a strongly realistic philosophy in Paris in the early XIIth century. After one of his students was very successful in argueing against Y's philosophy, Y retired to the abbey of St. Victor and was later made bishop of Châlons-sur-Marne.
  – Z was an archbishop of Canterbury of Italian descent, immediate predecessor of Anselm of Canterbury. At the Council of Vercelli in 1050, he defended the doctrine of *transsubstantiation* against Berengar of Tours.