Unifying Linguistic, Musical and Visual Processing

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What do Language, Music and Image have in common?

E.g.:

Language:
"List the sales of products in 2003"

Music:
\[\text{...}\]

Image:
\[\text{...}\]

At first sight very little...
How do we perceive Language, Music and Image?

Inherent to all forms of perception:

A *structuring process* in *groups*, *subgroups*, *sub-subgroups*, etc.

It is virtually impossible *not* to perceive structure

(People even assign structure to noise...)
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In music, grouping structure is typically represented as:
Grouping Structure in Music

The musical piece as a whole forms a group

A group consists of subgroups which are recursively built up out of smaller subgroups, up to the smallest unit (e.g. a pitch)

Grouping structure represents how parts combine into a whole
Grouping Structure in Language

Groups in language form a *tree structure* (Wundt 1880):

```
List the sales of products in 2003
```
Grouping Structure in Language

Groups in language form a *tree structure* (Wundt 1880):

Grouping structure in different representations (Chomsky 1956):

List the sales of products in 2003
Grouping Structure = Tree Structure

is equivalent (isomorphic) with:
Also Visual Groups form a Tree Structure

According to Wertheimer (1923) the visual input

is assigned the following structure:

Perceptual structuring forms the link between low-level segmentation and higher-level interpretation algorithms.
Perceptual Structure = Tree Structure

Relatively Uncontroversial:
There exists *one representation* for structural perception for all modalities
Perceptual Structure = Tree Structure

Relatively Uncontroversial:
There exists *one representation* for structural perception for all modalities

Very Controversial:
There exists *one model* that predicts the perceived structure in *language, music en vision*
Additional Problem: Perception is Ambiguous

The same input can be assigned several structures: ambiguity
Ambiguity is not just a problem

Average sentence from *Wall Street Journal* has more than one million different possible tree structures (Charniak 1999)

Adding semantics makes the problem even worse!
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"Any given sequence of notes is infinitely ambiguous, but this ambiguity is seldom apparent to the listener" (Longuet-Higgins 1987)

Humans perceive mostly just one grouping structure
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Humans perceive mostly just one grouping structure

> 96% agreement among subjects (language users)

**Language:** Penn Treebank  
**Music:** Essen Folksong Collection  
**Vision:** Nijmegen Visual Database
Historically, two competing principles for solving ambiguity in perception

   Preference for the simplest structure

2. Likelihood Principle (Helmholtz 1910...Suppes 1984, Charniak 2001)
   Preference for the most likely structure

Can these principles still inspire us?
The Dual Nature of Perception

These principles each play a *different* role in perception:

**Simplicity**: general preference for "economy", "least effort", "shortest derivation"

**Likelihood**: a memory-based bias due to previous experiences
The Dual Nature of Perception

These principles each play a *different* role in perception:

**Simplicity**: general preference for "economy", "least effort", "shortest derivation"

**Likelihood**: a memory-based bias due to previous experiences

**Hypothesis**: perceptual system strives for the *simplest* structure but in doing so it is influenced by the *likelihood* of previous structures
Possible Measures for Simplicity and Likelihood

**Simplicity**: number of "steps" to generate a tree structure

**Likelihood**: joint *probability* of the steps to generate a tree structure

We can compute this if we have a large, representative collection of tree structures for each modality (a "corpus")
Possible Measures for Simplicity and Likelihood

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**Data-Oriented Parsing model (DOP):**

*New input is analyzed and interpreted out of parts of previously perceived input*

(cf. CBR, Corpus-based NLP, EBL, ...)
Example of a DOP model for Language

Let's start with an extremely simple corpus:
A new sentence such as "She saw the dress with the telescope" is analyzed by **combining subtrees from the corpus**.
But there is also a "competing" analysis:

This analysis consists of two steps, and is therefore preferred according to the \textit{simplicity principle}: \textit{maximal similarity} with corpus.

But it is \textbf{not} preferred according to the \textit{likelihood principle}!
But there is also a "competing" analysis:

This analysis consists of two steps, and is therefore preferred according to the *simplicity principle*: maximal similarity with corpus.

But it is **not** preferred according to the *likelihood principle*!
Corpus

She wanted the dress on the rack.

She saw the dog with the telescope.
Corpus

S
  NP she
  VP wanted
    NP the dress
      PP on the rack
  VP
    NP
      PP
        P
          NP

Decompositie

S
  NP she
  VP
    NP
      PP
        P
          NP
            V
              NP
                PP
                  P
                    NP
                        V
                          NP
                            PP
                              P
                                NP
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                                                                                NP
                                                                                     NP
                                                                                       NP
                                                                                           etc.
she wanted the dress on the rack with the telescope.

she saw the dog with the telescope.

she saw the dress on the dog.

e tc.
DOP models are Stochastic Tree Grammars

By putting various constraints on STGs, we can instantiate:

- stochastic context-free grammars
- stochastic head-lexicalized grammars
- stochastic tree-adjoining grammars
- stochastic finite-state grammars
  etc...

We will focus on STSGs (Stochastic Tree Substitution Grammars)
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We will focus on STSGs (Stochastic Tree Substitution Grammars)

However, we have also developed DOP models for richer structures, such as *LFG, HPSG, Logical-Semantic* and *Discourse annotations*

(e.g. Bod & Kaplan 1998, 2003; Way 2003; Neumann 2003; Bod et al. 1996; Bod 1998)
Experiments with large corpora

Penn Treebank, Essen Folksong Collection:
Tens of thousands of analyzed sentences and folksongs
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**Simplest tree structure for string s:**
Minimize number $N$ of corpus subtrees in tree $T$

\[ T_{\text{best}} = \arg \min_T N(T \mid s) \]
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Maximize product of relative frequencies of subtrees $t_i$ in $T$

$$T_{best} = \arg \max_T P(T \mid s) = \arg \max_{t_1 \ldots t_n} \prod_i P(t_i \mid s)$$
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**Our best hypothesis so far:**
The perceptual system selects the *simplest* structure from the top of the distribution of *most probable* structures
The probability of:

\[ P(t) = \frac{|t|}{\sum_{t' : \text{root}(t') = \text{root}(t)} |t'|} \]

a subtree \( t \):

a derivation \( d = t_1 \circ \ldots \circ t_n \):

\[ P(t_1 \circ \ldots \circ t_n) = \prod_i P(t_i) \]

a parse tree \( T \):

\[ P(T) = \sum_d \prod_i P(t_{id}) \]

where \( t_{id} \) is the \( i \)-th subtree in derivation \( d \) that produces \( T \)
Computational Aspects of DOP

**Problem**: exponentially many subtrees in DOP / STSG

Can be solved by reducing DOP to an isomorphichic *Probabilistic Context-Free Grammar* or *PCFG*
Computational Aspects of DOP

**Problem:** exponentially many subtrees in DOP / STSG

Can be solved by reducing DOP to an isomorphic *Probabilistic Context-Free Grammar* or *PCFG*

Every node in every tree in corpus is assigned a unique number:

- \( A@k \) denotes node at address \( k \) where \( A \) is nonterminal of that node

- A new nonterminal is created for each node in the training data: \( A_k \)
Sketch of PCFG reduction of DOP (1)

Consider a node A@j of the following form in STSG/DOP:

```
A@j
  B@k  C@l
```
Sketch of PCFG reduction of DOP (1)

Consider a node $A_{@j}$ of the following form in STSG/DOP:

```
  A_{@j}
   / \   /
  B_{@k} C_{@l}
```

There are $b_k$ non-trivial subtrees headed by $B_{@k}$ plus trivial case where left node is simply $B$. 
Sketch of PCFG reduction of DOP (1)

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   /
  B@k C@l
```

There are $b_k$ non-trivial subtrees headed by $B@k$ plus trivial case where left node is simply $B$.

Thus $b_k + 1$ different possibilities on the left branch

Similarly, $c_l + 1$ possibilities on the right branch

Thus, $a_j = (b_k + 1)(c_l + 1)$ possible subtrees headed by $A@j$
Sketch of PCFG reduction of DOP (2)

There is a PCFG with the following property (Bod 2003; Goodman 2003):

for every subtree in training corpus headed by $A$, the PCFG will generate an isomorphic subderivation with probability $1/a$

\[
\begin{align*}
A_j \rightarrow BC & \quad (1/a_j) \quad A \rightarrow BC & \quad (1/a) \\
A_j \rightarrow B_kC & \quad (b_k/a_j) \quad A \rightarrow B_kC & \quad (b_k/a) \\
A_j \rightarrow BC_l & \quad (c_l/a_j) \quad A \rightarrow BC_l & \quad (c_l/a) \\
A_j \rightarrow B_kC_l & \quad (b_kc_l/a_j) \quad A \rightarrow B_kC_l & \quad (b_kc_l/a)
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\]

Rather than using all subtrees, we can use a "compact" PCFG!
Sketch of PCFG reduction of DOP (3)

- Dynamic programming algorithm known as Viterbi bottom-up search computes most probable derivation for input string
- Same algorithm can be used to compute shortest derivation (i.e. simplest tree) by assigning each subtree equal probability
Sketch of PCFG reduction of DOP (3)

- Dynamic programming algorithm known as *Viterbi bottom-up search* computes *most probable derivation* for input string

- Same algorithm can be used to compute *shortest derivation* (i.e. simplest tree) by assigning each subtree equal probability

  Thus both *likeliest* and *simplest* tree are efficiently computed

  Next, we can compute the simplest among the *n* likeliest trees also by *Viterbi n best search*
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- Dynamic programming algorithm known as Viterbi bottom-up search computes most probable derivation for input string.

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  Thus both likeliest and simplest tree are efficiently computed.

  Next, we can compute the simplest among the $n$ likeliest trees also by Viterbi $n$ best search.

- Other work has proposed different computational solutions:

  Voted Perceptron (Collins), Tree Kernels (Bod, Duffy), MaxEnt (Sima'an), MDL (Bonnema), E-M (Prescher)...
Test Domains

- **Linguistic test domain:**
  
  Wall Street Journal (WSJ) corpus in the Penn Treebank: 50,000 manually analyzed sentences
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• **Musical test domain:**
  Essen Folksong Collection (EFC): 20,150 melodically analyzed western folksongs:
    • *Pitches*: numbers from 1 to 7
    • *Duration indicators*: underscore (\_) or a period (.) *after* the numbers
    • *Octave position*: plus and minus signs (+,-) *before* the numbers
    • *Chromatic alterations*: "#" or "b" *after* the numbers
    • *Pauses*: 0, possibly followed by duration indicators
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• **Visual test domain:** see later
Example from Essen Folksong Collection

#4551: Schneckhaus Schneckhaus stecke deine Hörner aus
    (German children song)

5_3_5_3_1234553_1234553_12345_3_12345_3_553_553_553_65432_1_
Example from Essen Folksong Collection

#4551: Schneckhaus Schneckhaus stecke deine Hörner aus
   (German children song)

5_3_5_3_1234553_1234553_12345_3_12345_3_553_553_553_65432_1_

   Grouping structure according to Essen Folksong collection:

   ((5_3_5_3_) (1234553_) (1234553_) (12345_3_) ( 12345_3_) (553_553_)
    (553_65432_1_))

NB: linguistic phrase structure does not predict musical phrase structure!
Preprocessing the Essen Folksong Annotations

• We automatically added three basic labels to the phrase structures:

  "S" to each whole song
  "P" to each phrase
  "N" to each note

• In this way, we obtain conventional tree structures that can be used by DOP/STSG, or its isomorphic PCFG
Examples of some simple musical trees
Experimental Evaluation

Corpora are randomly divided into 10 training/test set splits
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Test 1: Simplicity-Likelihood-DOP (SL-DOP)

Selects simplest structure from among n likeliest structures
Experimental Evaluation

Corpora are randomly divided into 10 \textit{training/test set splits}

\textbf{Test 1: Simplicity-Likelihood-DOP (SL-DOP)}

Selects \textit{simplest} structure from among \textit{n} likeliest structures

\textbf{Test 2: Likelihood-Simplicity-DOP (LS-DOP)}

Selects \textit{likeliest} structure from among \textit{n} simplest structures
# Scores of SL-DOP & LS-DOP

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<tr>
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**Same** model obtains **maximal** scores for both language and music.

Perceptual system strives for **simplest** analysis, but "searches" only among the **most likely** analyses (see Schaefer et al. 2004 for psychological experiments)
Comparison with other work

Language: DOP outperforms Collins, Charniak, Ratnaparkhi on WSJ

→ non-head dependencies can only be covered by subtrees without (lexical) restrictions
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By using largest possible subtrees (simplest analysis) which occur most frequently, DOP takes into account more dependencies
Example of non-headword dependency (ATIS corpus)

E.g.: Show the nearest airport to Denver

- non-head modifier *nearest* predicts the correct PP-attachment
- Example from WSJ: *BA carried more people than cargo in 1988*
Example of "jump phrase" (EFC)

Folksong K0690:

\[(3\_2\_11-5) (-5332211-5) (-512314\_2) (\ldots)\]

- Gestalt principles predict "wrong" phrases on large intervals:

\[(3\_2\_11-5-5) (332211-5-5) (12314\_2) (\ldots)\]

- Parallelism, meter & harmony reinforce same "wrong" predictions!

- Many phrases reflect idiom-dependent pitch contours which cannot be predicted by rules, but only by "patterns" (Cf. Huron 1996)
The importance of large subtrees

• Large subtrees may be statistically significant though they are linguistically and musically redundant

• Continuum between "regular phrases" (rules) and "idiomatic phrases" (patterns) both in language and music

• DOP can capture the full gradience between rules and patterns
How can we apply this to Visual Structures?

Structured visual databases are still too small (<300) to get statistically significant results.
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What are the primitive elements in visual perception?

In Nijmegen Visual Database:  
*line segments*, *angles*, *a.o.*

"Syntactic" categories: *symmetry* \( (S) \), *alternation* \( (A) \), *iteration* \( (I) \).

Of course, we only deal with medium-level computer vision in this way.
Experiments support SL-DOP, but not statistically significant.
DOP is used in various AI applications

- **Structural language models for speech** (Bod 1998, 2000; Chelba 1998)

  \[
  \text{argmax}_W P(W | A) = \text{argmax}_W \sum_T P(W, T | A)
  \]

- **Statistical machine translation** (Hearne & Way 2004; Poutsma & Bod 2003)

  \[
  \text{argmax} P(\text{Translated sentence} | \text{Source sentence})
  \]

- **Musical tempo tracking systems** (Zaanen, Honing & Bod 2004)

  \[
  \text{argmax} P(\text{Temporal structure} | \text{Acoustic input})
  \]

- **Interactive spoken dialog systems** (Bod 1999; Scha et al. 1999), used by OVIS

  \[
  \text{argmax} P(\text{Interpretation, Word string} | \text{Acoustic signal})
  \]
Example of OVIS annotation used in spoken dialog
MP(d1;d2) VPd1.d2

S d1.d2

ik

V wants

wil

PER user

P destination.place

van

van

naar

NP town.venlo
town.almere

=
How far do exemplar-based models stretch?

- Problem solving with exemplar-based model such as DOP/STSG?
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- Exemplar-based reasoning has been proposed as early as Thomas Kuhn in his account on normal science (in his "Structure of ...")

  "Scientists solve problems by modeling them on previous problem-solutions" (Kuhn 1962)
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• Problem-solutions in physics can be represented by derivation trees -- though they do not represent grouping structure
Example of derivation tree in classical mechanics

Derivation of planet's mass from a satellite's orbit using Newton's laws

\[ F = ma \]
\[ a = \frac{v^2}{r} \]
\[ F = \frac{mv^2}{r} \]
\[ v = \frac{2\pi r}{P} \]
\[ F = \frac{4\pi^2 mr}{P^2} \]
\[ F = \frac{GMm}{r^2} \]

\[ 4\pi^2 mr/P^2 = GMm/r^2 \]

\[ M = \frac{4\pi^2 r^3}{GP^2} \]

A tree describes the steps from higher-level laws to the solution (formula)
Subtrees can be reused to solve new problems

\[ F = ma \]
\[ a = \frac{v^2}{r} \]
\[ F = \frac{mv^2}{r} \]
\[ v = \frac{2\pi r}{P} \]
\[ F = \frac{4\pi^2 m r}{P^2} \]
\[ F = \frac{GMm}{r^2} \]
\[ \frac{4\pi^2 m r}{P^2} = \frac{GMm}{r^2} \]
Deriving Kepler's third law by this subtree

\[ F = ma \quad a = \frac{v^2}{r} \]

\[ F = \frac{mv^2}{r} \quad v = \frac{2\pi r}{P} \]

\[ F = \frac{4\pi^2mr}{P^2} \quad F = \frac{GMm}{r^2} \]

\[ \frac{4\pi^2mr}{P^2} = \frac{GMm}{r^2} \]

\[ \frac{r^3}{P^2} = \frac{GM}{4\pi^2} \]

We only need to solve the last equation of the previous subtree for \( \frac{r^3}{P^2} \)
Often we need to combine two or more subtrees (by term rewriting)

\[ F = ma \quad a = \frac{v^2}{r} \]

\[ F = \frac{gm^2}{r^2} \quad \Rightarrow \quad \frac{gm^2}{r^2} = \frac{GMm}{r^2} \]

\[ v = \sqrt{\frac{GM}{r}} \]
Derivation trees in fluid mechanics

E.g. derivation of *orifice system* from Bernoulli involves an ad hoc correction coefficient ($C_d$)

\[ \Sigma E = \text{constant} \]

\[ \rho g z_1 + \rho v_1^2/2 + p_1 = \rho g z_2 + \rho v_2^2/2 + p_2 \]

\[ p_1 = p_2 \]
\[ v_1 = 0 \]
\[ z_1 - z_2 = h \]

\[ v = \sqrt{2gh} \]

\[ Q(\text{theoretical}) = vA \]

\[ Q(\text{theoretical}) = A\sqrt{2gh} \]

\[ Q(\text{actual}) = C_d Q(\text{theoretical}) \]

\[ Q(\text{actual}) = C_d A\sqrt{2gh} \]
Derivation tree for a *weir* (*dam*) can still be derived by subtrees from orifice system that include the ad hoc correction

\[ \Sigma E = \text{constant} \]

\[ \rho g z_1 + \rho v_1^2/2 + p_1 = \rho g z_2 + \rho v_2^2/2 + p_2 \]

\[ v = \sqrt{2gh} \]

\[ p_1 = p_2 \]
\[ v_1 = 0 \]
\[ z_1 - z_2 = h \]

\[ Q(\text{theoretical}) = vA \]

\[ Q(\text{theoretical}) = \int v dA \]
\[ dA = Bdh \]

\[ Q(\text{theoretical}) = B \sqrt{2g} \int \sqrt{h} dh \]

\[ Q(\text{theoretical}) = (2/3)B \sqrt{2g} h^{3/2} \]

\[ Q(\text{actual}) = C_d Q(\text{theoretical}) \]

\[ Q(\text{actual}) = (2/3)C_d B \sqrt{2g} h^{3/2} \]
Simplicity and Likelihood also in problem solving

- Prefer largest possible derivational chunks, such that minimal recourse to additional derivational steps is needed
Simplicity and Likelihood also in problem solving

- Prefer largest possible derivational chunks, such that minimal recourse to additional derivational steps is needed
- Prefer more frequently occurring chunks: reflects usefulness
Simplicity and Likelihood also in problem solving

- Prefer largest possible derivational chunks, such that minimal recourse to additional derivational steps is needed.
- Prefer more frequently occurring chunks: reflects usefulness.
- $P(Derivation-tree \mid Phenomenon)$ can be computed in a Bayesian way as in language and music, given a corpus of "exemplars".

We have just received an NWO grant for "Exemplar-Based Explanation" (one postdoc and one phd student).
Conclusions

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Conclusions

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• AI should aim at developing general models for (each level of) cognition rather than particularist models for each cognitive task separately
  
  there are autonomous levels of explanation, but without striving for underlying models AI becomes a plethora of disparate algorithms