UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2005/2006; 1st Semester
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Homework Set # 7 *Deadline:* November 1st, 2005 (**two weeks!**)

Exercise 21 (total of seven points).

We are considering two new systems of dialogic logic: In the first one, called **strictly constructive**, we restrict the proponent in a way that he also can only react to the last move of the opponent and denote the corresponding semantic relation by $\models_{\rm sc}$. In the second one, called **liberal**, $\models_{\rm lib}$, we liberalize the opponent so that he also can react to all prior moves of the proponent.

(1) Give formal definitions (in the style of the lecture, giving explicitly the rules for the two players) for $\models_{\rm sc}$ and $\models_{\rm lib}$ (½ points each).
(2) Prove that $\models_{\rm lib} \varphi$ holds for no formula φ (2 points).
(3) Find two different formulas φ such that $\models_{\rm sc} \varphi$ and give dialogue proofs for them (1 point each).
(4) Find a formula φ such that $\models_{\rm dialog} \varphi$ but not $\models_{\rm sc} \varphi$. Give proofs of both claims (1 point each).

Exercise 22 (total of four points).

Give dialogue proofs of the following formulas in $\models_{\rm cl}$ (1 point each):

- $\neg\neg\neg p \rightarrow \neg p$,
- $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$.

For both formulas, decide whether they are valid in $\models_{\rm dialog}$ and give a dialogue argument for or against your claim (1 point each).

Exercise 23 (total of six points).

In this exercise, we consider the systems of *positio* as described by Walter Burley and Roger Swyneshed. If a *positum* φ^* is given and φ_k (for $0 \leq k \leq n$) are proposed sentences of the **Opponent**, we let $\Phi_k^{\rm Burley}$ be the set of **"currently accepted truths"** according to Burley's system on the basis of the sequence $\langle \varphi^*, \varphi_0, \ldots, \varphi_n \rangle$.
Prove the following properties of the two systems:

(1) If the *positum* φ^* is consistent, then for all $k \leq n$, the set $\Phi_k^{\rm Burley}$ is a consistent set (3 points).
(2) If the *positum* φ^* is consistent and $k < \ell \leq n$ with $\varphi_k = \varphi_\ell$, then the **Respondent** in a Swyneshed-style *positio* will give the same answer in steps k and ℓ of the *obligatio* (3 points).

Exercise 24 (total of five points).

We are considering a system reminiscent of Leibniz' attempts to arithmetize language. In the lecture, we introduced a system based on the divisor structure of the natural numbers, but this system was too simple as it didn't allow proper discussion of negative statements. Therefore, we add a number that should take care of the negative statements to the system. (The rough idea is: If 2 is *animal*, 3 is *rationalis* and 7 is *asinarius* ("donkey-like"), then $\langle 6, 7\rangle$ would represent *homo* (to preclude the option of constructing a *homo asinarius*) and $\langle 14, 3\rangle$ would represent *asinus* (to preclude the option of constructing an *asinus rationalis*.)

Formally: Call a pair $X := \langle \mathrm{p}_X, \mathrm{n}_X\rangle$ a **pseudo-Leibniz predicate** (**PLP**) if p_X and n_X are both positive natural numbers ≥ 2. We write $n|m$ for "n divides m" (*i.e.*, there is a $k \geq 1$ such that $nk = m$) and $n \perp m$ for "n and m are coprime" (*i.e.*, if $k|n$ and $k|m$, then $k = 1$). We define the following semantics for categorical propositions using PLPs:

$$
\begin{aligned}
X\mathrm{a}Y &:\equiv \mathrm{p}_X|\mathrm{p}_Y \,\&\, \mathrm{p}_Y \perp \mathrm{n}_X \\
X\mathrm{i}Y &:\equiv \exists k \geq 1(\mathrm{p}_X|k\cdot \mathrm{p}_Y \,\&\, k\cdot \mathrm{p}_Y \perp \mathrm{n}_X) \\
X\mathrm{e}Y &:\equiv \forall k \geq 1(\neg(\mathrm{p}_X|k\cdot \mathrm{p}_Y) \;\vee\; \neg(k\cdot \mathrm{p}_Y \perp \mathrm{n}_X))
\end{aligned}
$$

In this semantics, **Barbara** can be expressed as:

$$\forall X, Y, Z\,((\mathrm{p}_X|\mathrm{p}_Y \,\&\, \mathrm{p}_Y|\mathrm{p}_Z \,\&\, \mathrm{p}_Y \perp \mathrm{n}_X \,\&\, \mathrm{p}_Z \perp \mathrm{n}_Y) \to \mathrm{p}_X|\mathrm{p}_Z \,\&\, \mathrm{p}_Z \perp \mathrm{n}_X)\,.$$

(1) Define a semantics for $X\mathrm{o}Y$ such that this is contradictory to $X\mathrm{a}Y$ (½ point).
(2) Give an example of a PLP that shows that **Barbara** is not valid with this semantics (2 points).
(3) Prove that **Celarent** is valid with this semantics (2½ points).