



Core Logic

2005/2006; 1st Semester
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Homework Set # 5

Deadline: October 11th, 2005

Exercise 13 (total of seven points).

Let W be a set of states and $R \subseteq W \times W$ an accessibility relation. We say “state v is conceivable by anyone in state w ” for wRv . Let X be a set of objects, and $E \subseteq W \times X$ a relation. We say “object x exists in state w ” for wEx . For each $w \in W$, we have a strict linear order $<_w$ of X , and we say “in state w , object x is better than object y ” for $y <_w x$.

We call $\langle W, R, X, E, \langle <_w ; w \in W \rangle \rangle$ an **ontological frame** if R is reflexive (i.e., w is conceivable by anyone in state w), and the following principle “*Existence is better than nonexistence*” (EBN) holds:

(EBN) For all x, y and w , if wEx and $\neg wEy$, then $y <_w x$.

The central argument of Anselm’s ontologic proof is “if something is such that nothing better can be conceived, then it must exist”. Formulate this argument in the language of ontological frames and prove it (4 points).

Given an example of an ontological frame where there is no object “such that nothing better can be conceived” (3 points).

Exercise 14 (total of seven points).

Read the text

Paul Vincent Spade, *Why Don’t Mediaeval Logicians Ever Tell Us What They’re Doing? Or, What Is This, A Conspiracy?*, preprint 2000

(PDF file on the course webpage) and answer the following questions:

- (1) What are Spade’s four ‘exhibits’ for the thesis that “*we simply don’t know what is going on*”? (¼ point each)
- (2) According to Spade, what does Richard Billingham mean by “immediate terms”? (2 point)
- (3) Spade is not concerned that Billingham’s proof of “A man runs” doesn’t prove anything we didn’t know before. What is it that causes Spade trouble with Billingham’s example? (2 points)
- (4) Would Spade subscribe to the following statements (1 point each):
 - (a) ‘We don’t understand medieval logic because we don’t have a full grasp of the underlying medieval philosophy.’
 - (b) ‘For the theories mentioned in the four exhibits, the historically earliest texts are lost, and this is the main reason why we don’t understand what is going on.’

Exercise 15 (total of eight points).

Many medieval authors think of disjunction as an operator on finite sets of sentences and define $\text{MD}(A_1, \dots, A_n)$ to be true if exactly one of the A_i is true.

If f is a binary truth function (i.e., a function from $\{0, 1\} \times \{0, 1\}$ to $\{0, 1\}$), we can use it to recursively define n -ary truth functions by

$$\begin{aligned} f_2(A, B) &:= f(A, B) \\ f_{n+1}(A_0, \dots, A_n) &:= f(f_n(A_0, \dots, A_{n-1}), A_n). \end{aligned}$$

We say that an n -ary truth function g is **induced by** f if $g = f_n$.

- (1) Show that medieval disjunction MD is not induced by any binary truth function (4 points).
- (2) Let h be exclusive disjunction (i.e., $h(0, 0) = h(1, 1) = 0$ and $h(0, 1) = h(1, 0) = 1$). Prove that $h_{2n}(A_0, \dots, A_{2n-1})$ is true if and only if an odd number of the A_i is true and that $h_{2n+1}(A_0, \dots, A_{2n})$ is true if and only if an even number of the A_i is true (4 points).