Formalism I

Edgar Andrade / Tijmen Daniels

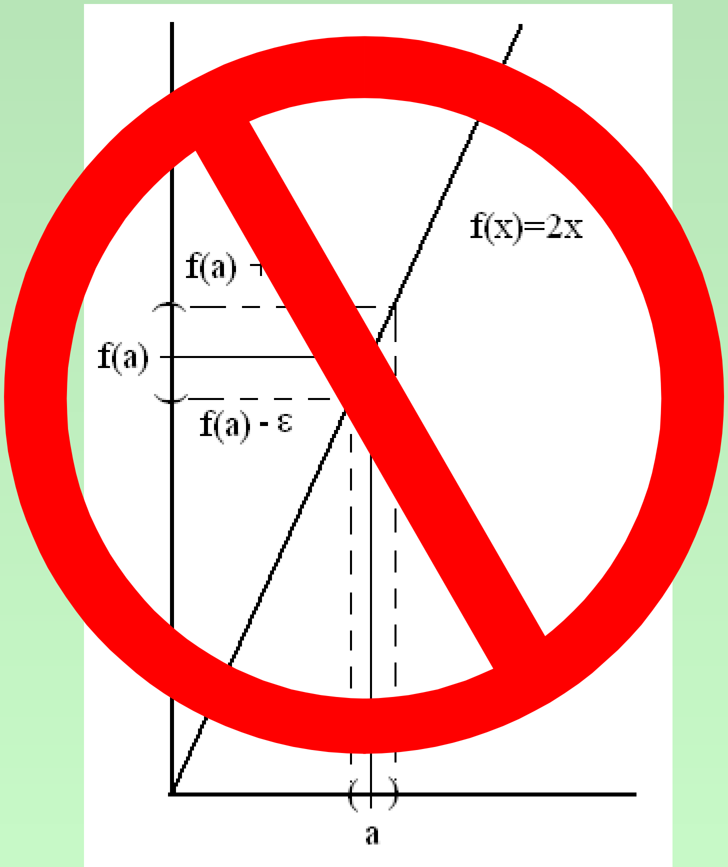
Philosophy of Mathematics

ILLC - Master of Logic

Introduction

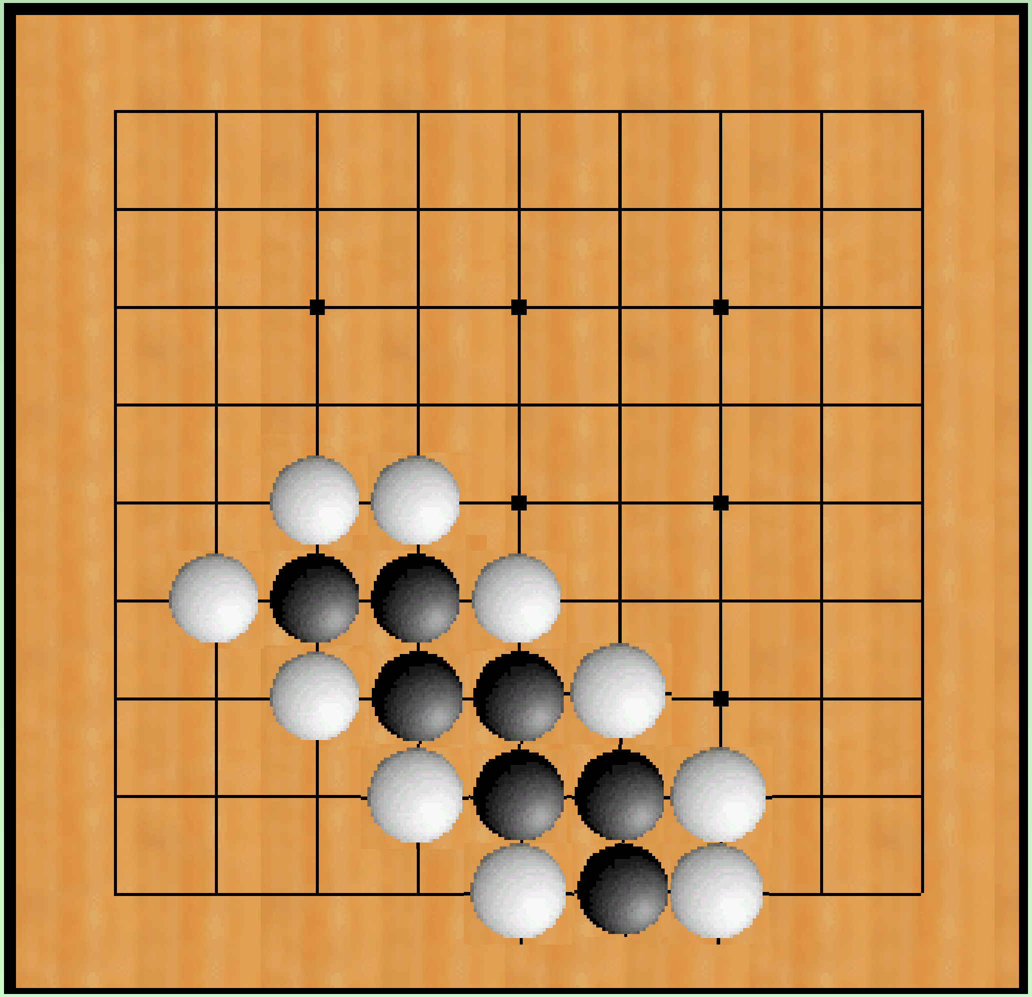
- A general characteristic, with few exceptions, in the systems presented thus far is that they have only considered geometry and arithmetic in their analysis. Realism in those branches suggests easily. However, this is not quite so in other branches, like complex analysis, or topology, or abstract algebra, etc..
- What are imaginary numbers, really? A common response to such dilemmas is to retreat to formalism. The mathematician asserts that symbols for complex numbers, for example, are to be manipulated according to (most of) the same rules as real numbers, and that is all there is to it.

Continuous functions



- Claim: f(x) is continuous at a.
- Let ε>0. Define δ= ε/2
- If | x-a | < δ, then
- | x-a | < ε/2, then
- 2| x-a | < ε, then
- | 2x-2a | < ε
- Easy game!

Games



- It's black's turn.
- Claim: Black group has no chances of survival.

Black group has no liberties and therefore is dead!

Games and Mathematics

- Close analogy as to both activities.
- Understanding them depends only on understanding their rules.
- It doesn't make much sense to ask about the ontology either of go or of mathematics.
- The structures in mathematics are unique *modulo isomorphism*.
  - The series 0,1,2,3,... Obey the same rules with regards to addition than the series 0,2,4,6,...

Basic Formalisms

- Main names:
  - E. Heine (1821-1881),
  - Johannes Thomae (1840-1921)
- Term Formalism:
  - The term formalist identifies the entities of mathematics with their names.
  - The entity refered to by '8+2i' is just the symbol '8+2i'.
- Games Formalism:
  - The symbols are meaningless -they don't refer to anything.
  - The content of mathematics is exhausted by the rules for operating with its language.

Basic formalisms (2)

- Type-token distinction
  - Tokens are physical objects, and as such can be destroyed and created at will. (e.g. The shadow on the board following the dash - c)
  - Types are the abstract forms of tokens. (e.g. The consonant “c”)
- Ontological commitments
  - Term formalism: Mathematics is about types.
  - Game formalism: No ontology.
- How is mathematics know? What is mathematical knowledge?
  - Term formalism: It is knowledge of how the characters are related to each other, and how they are to be manipulated in the mathematical practice.
  - Game formalism: It is knowledge of the rules of the game.

Problems with Term Formalism

- The intuitive (and the logical) meaning of ‘=’ is the identity.
- Problem: It cannot be identity between types, because the types of ‘5+7’ and ‘6+6’ are not the same and yet ‘5+7=6+6’ should hold.
- Solution: ‘A=B’ says that the symbol corresponding to A is intersustitutable with the symbol corresponding to B in any mathematical context.

Problems with Term Formalism (2)

- What about the real numbers? There are TOO many real numbers that don’t have a name yet they are mathematical entities (don’t try to move to the decimal expansion of real numbers because this expansion is infinite).
- Even though tokens are not abstract entities, types are. So we end up claiming the existence of abstract entities. What is the advantage of this instead of claiming the existence of numbers from the outset?
- Moreover, in what sense the fundamental theorem of algebra can be said to be about symbols?

Game Formalism

- The terms ‘language’ and ‘symbols’ could be misleading. No ‘aboutness’ in Game Formalism.
- If mathematics has meaning, it is extraneous to mathematics itself. Meaning is at most an heuristic element.
- What is the ontological status of rules? (Big issue about rule following. Cf. Philosophical Investigations)
- Why are the mathematical games so useful in sciences?

Frege’s Critique

- Frege: ‘To be sure, there is an important difference between arithmetic and chess. The rules of chess are arbitrary, the system of rules of arithmetic is such that by means of simple axioms the number can be referred to manifolds and can thus make important contributions to our knowledge of nature.’(1898: SS1-11)
- Problems for Frege:
  - Ontology is easy for natural numbers, but what about the ontology for topology or for complex analysis.
  - Reference doesn’t explain aplicability, since it is not clear how these objects fit in the causal order.