Termistic logic (1).

Moving from analysis of meaning in words (what does *homo* mean?) to analysis of meaning of terms in phrases (what part of the meaning of *homo* is responsible for the fact that “*omnis homo mortalis est*” is true?).

- Syllogistics doesn’t analyse the truth-status of categorial propositions any further.
- Linguistic analysis (predication vs non-predication) at the basis of the theory of categories.
- Grammar investigated the meaning of single words (outside of the context of propositions).
- Origins in the school of Chartres (c.1030): ‘contextual approach’ (de Rijk, 1967).
Subtle questions.

- Compare “homo est animal”, “homo est species”, and “homo est disyllabum”. In each of the cases, the meaning of homo is slightly different.

- What do qualifiers do with meanings? If I go from “omnis homo est philosophus” to “paene omnis homo est philosophus”, how does the explanation for the meaning change?
Aristotle / Boëthius: Terms *signify* by establishing an understanding. **Signification** has a causal component.

**Triad**: written / spoken / mental language. Written language signifies spoken language, spoken language signifies mental language, mental language signifies the things.

*Significatio* is determined by *impositio*, *i.e.*, the word’s original application (baptism).

Priscian: *proprium est nominis significare substantiam cum qualitate*. *philosophus* signifies “human with the quality of being a philosopher”.

Terms that signify on their own: *categoremata*.

Terms that only *consignify*: *syncategoremata*. 
Syncategoremata.

Grammarians’ definition. A term is a categorema if it can be the subject or the predicate of a proposition. Other meaningful terms are syncategoremata.

Example 1. *Socrates currit*.

Example 2. *Socrates non currit*.

Logicians’ definition. An incomplete list of about fifty words that are discussed as syncategorematic. Among them are words like *omnis*.

Important syncategoremata: *et, ut, cum, vel, omnis, uterque*...
Suppositio (1).

An analysis of the meaning of terms in propositions: *Suppositio* as a theory of reference.

**Situation 1.**
- Under what conditions is *omnis homo philosophus est* true?
- If *philosophus* supposits for every instance of *homo* (*suppositio mobilis*).

**Situation 2.**
- Under what conditions is *omnis homo praeter Socratem philosophus est* true?
- If *philosophus* supposits for every instance of *homo* except for Socrates.
- *Instantiation: Aristoteles homo est. Aristoteles praeter Socrates philosophus est* (*suppositio immobils*).
An aside.

Latin doesn’t have an indefinite article.

- *Homo est philosophus.*
- A man is a philosopher.
- (Some man is a philosopher.)
- *Aliquis homo est philosophus.*

The medievals didn’t use quotation marks.

- *Homo est disyllabum.*
- ‘Human’ is bisyllabic.
Suppositio (2).

Situation 3.

- Under what conditions is *homo est disyllabum* true?
- If *disyllabum* supposits for every instance of *homo*. (But here, *homo* is a singular term standing for ‘*homo*’).
- *Flawed instantiation*: Aristoteles *homo est*. Aristoteles *disyllabum est*. (*suppositio materialis*).

**Consequences for logic**: Whether conversion rules can be applied depends on the type of supposition in the proposition.

\[
\begin{align*}
  \text{homo est disyllabum.} \\
  \text{aliquis homo est disyllabum.} \\
  \text{aliquis disyllabum est homo.} \quad \text{(simple conversion)} \\
  \text{disyllabum est homo.}
\end{align*}
\]

Bisyllabic is a man.
Suppositio (3).

Types of *suppositio* (Spade 1982):

- *suppositio impropria*.
- *suppositio propria*.
  - *suppositio materialis*.
  - *suppositio formalis*.
    - *suppositio discreta*.
    - *suppositio simplex*.
    - *suppositio personalis*.
      - *suppositio determinata*.
      - *suppositio confusa tantum*.
      - *suppositio mobilis*.
      - *suppositio immobils*.

Paul Vincent Spade, Thoughts, Words and Things: An Introduction to Late Mediaeval Logic and Semantic Theory, *preprint*

[http://www.pvspade.com/Logic/](http://www.pvspade.com/Logic/)
Suppositio (4).

- What makes *Aristoteles academicus erat* true?

- **Attempt 1.** If *academicus* supposits for *Aristoteles*. But if *academicus* supposits for *Aristoteles*, then *Aristoteles academicus est* is true.

- **Attempt 2** (modern reading). If there was a point in the past when *academicus* supposited for *Aristoteles*.

- Medieval theory: ampliation and restriction: *si terminus communis verbo de praeterito supponeret, posset supponere pro non-enti, ut hoc homo cucurrit verum est pro Caesare* (William of Shyreswood, *Introductiones*).

- In general: the predicate determines the type of *suppositio* and whether *ampliation* has to be used in order to determine the truth conditions.
Suppositio (5).

Still: Even ampliation only allows to move back and forth in time, or along other modal accessibility relations (conceivability, possibility etc.).

Omnis *chimaera est chimaera* is false regardless of ampliation since there was never and will never be an instantiation of “*chimaera*” that *chimaera* could supposit for.

St. Vincent Ferrer (1350-1419), *De suppositionibus dialecticis* (1372). “*rosa est odorifera*” is true even if there are no roses and never have been roses.
Via moderna (1).


- *Quaestiones Victorinae* (school of William of Champeaux, 1100-1150).

- Golden Age of Terminist Logic: 1175-1250.
  - *Ars Meliduna* (1170-1180).
  - *Tractatus Anagnini* (1200-1220).
  - Petrus Hispanus (Pope John XXI.; c.1205-1277): *Summulae Logicales* (c.1230-1240).
Via moderna (2).

- **Oxford School.**
  Influenced by the Parvipontani.
  
  *Main representative.* William of Shyreswood.

- **Paris School.**
  
  *Main representative.* Petrus Hispanus.

- **Geoffrey of Hapshall (c.1270).**

- **Modists (XIIIth and XIVth century).**
  
  “speculative grammar” based on *modi significandi* and *De anima*.

  - Boëthius of Dacia (d.1290)
  - Pierre d’Auvergne (d.1303)
  - Martin of Dacia (d.1304)
  - Thomas of Erfurt (c.1330)
  - Johannes Aurifaber (c.1330)
Via moderna (3).

Via Antiqua.

- *logica vetus* (in particular, the *Categoriae*).
- Thomistic realism.

John Wyclif (c.1330-1384).

Girolamo Savonarola (1452-1498).

Via Moderna.

- *logica nova*.
- Semantical analysis (scholastics).
- Nominalism.

The Terminists.

The Modists.

Walter Burley (c.1275-1344).

William Ockham (c.1295-1349).

XIVth and XVth century. Philosophy sharply divided into *via antiqua* and *via moderna*. 
Fallacies: *secundum quid et simpliciter*.

Around 1120, Boëthius’ translation of the *Sophistici Elenchi* is rediscovered. Aristotelian discussions of fallacies.

The Oathbreaker:

- **Oath.** I shall never leave Rome. I shall become an oathbreaker.

- **Fact.** I have left Rome.

*Argument.* Since I have left Rome, I broke my oath. Since I have broken, I have kept my oath. I am an oathbreaker and an oathkeeper at the same time. I am an oathbreaker and an oathkeeper.

*secundum quid et simpliciter*

- **simpliciter.** An oathbreaker is a person who breaks at least one oath.

- **secundum quid.** An oathkeeper is a person who keeps the oath.
Insolubles (1).

The most famous insoluble: the Liar.

This sentence is false.

\[ \varphi : \varphi \text{ is false.} \]

In the early literature on insolubles, there are five solutions to this paradox:

- secundum quid et simpliciter.
- transcasus.
- Distinction between the exercised act and the signified act.
- restrictio.
- cassatio.
Insolubles (2).

- *secundum quid et simpliciter.*
- Mentioned by Aristotle (*Sophistici Elenchi*, 180b2-3).
Insolubles (2).

- *secundum quid et simpliciter.*

  **Solution.** Unclear.

- *transcasus.*
  - Derives from the Stoic *metaptosis*: differing truth-values over time.
  - When I say “I am speaking a falsehood” I am referring to what I said immediately preceding to that sentence.
  - If I didn’t say anything before that, then the sentence is just false.
Insolubles (2).

- *secundum quid et simpliciter.*
  
  **Solution.** Unclear.

- *transcasus.*
  
  **Solution.** The Liar sentence is false.

- Distinction between the exercised act and the signified act.
  
  - Johannes Duns Scotus, *Questiones.*
  - The exercised act of the liar is “speaking the truth”.
  - The signified act of the liar is “speaking a falsehood”.
  - The liar expresses something which is not the truth, so it is false.
Insolubles (2).

- *secundum quid et simpliciter.*
  **Solution.** Unclear.

- *transcasus.*
  **Solution.** The Liar sentence is false.

- Distinction between the exercised act and the signified act.
  **Solution.** The Liar sentence is false.

- *restrictio.*
  - The *restringentes* do not allow assignment of truth-values to sentences with self-reference.
  - Not only the Liar, but also the following insoluble: $\varphi : \psi$ is false. $\psi : \varphi$ is false (linked liars)
Insolubles (2).

- secundum quid et simpliciter.
  Solution. Unclear.

- transcasus.
  Solution. The Liar sentence is false.

Distinction between the exercised act and the signified act.
Solution. The Liar sentence is false.

- restrictio.
  The restringentes do not allow assignment of truth-values to sentences with self-reference.
  Not only the Liar, but also the following insoluble: $\varphi : \psi$ is false. $\psi : \varphi$ is false...
  ... and ... “This sentence has five words.”
Insolubles (2).

- **secundum quid et simpliciter.**
  **Solution.** Unclear.

- **transcasus.**
  **Solution.** The Liar sentence is false.

- Distinction between the exercised act and the signified act.
  **Solution.** The Liar sentence is false.

- **restrictio.**
  **Solution.** The Liar sentence does not have a truth value.

- **cassatio.**
  
  - If you are uttering an insoluble, you are saying nothing.
  
  - Therefore an insoluble has the same truth value as the empty utterance: none.
Insolubles (2).

- *secundum quid et simpliciter.*
  Solution. Unclear.

- *transcasus.*
  Solution. The Liar sentence is false.

- Distinction between the exercised act and the signified act.
  Solution. The Liar sentence is false.

- *restrictio.*
  Solution. The Liar sentence does not have a truth value.

- *cassatio.*
  Solution. The Liar sentence does not have a truth value.
Insolubes (3).

- The most productive era in the theory of insolubles was from 1320 to 1350.
  - Thomas Bradwardine (c.1295-1349).
  - Roger Swyneshed (mid XIVth century).
  - William Heytesbury (c.1310-1372).
  - Gregory of Rimini (mid XIVth century).
  - John Wyclif (c.1330-1384).
  - Peter of Ailly (*Petrus de Alliaco*; 1350-1420).
Bradwardine.

Thomas Bradwardine (c.1295-1349).

- *Insolubilia*: 1321-1324.
- *Adverbial Theory of propositional signification* (Spade).

Every sentence signifies that it is true.

A sentence is **true** if and only if everything that it signifies is true (*sicut est*). A sentence is **false** if and only if there is something that it signifies which is false (*aliter quam est*).

The Liar sentence signifies that it is false.

\[
\varphi : \varphi \text{ is false} \\
\varphi \text{ is false} \quad \text{signifies} \quad \varphi \text{ is true} \quad \text{signifies}
\]
Swyneshed.

Roger Swyneshed (mid XIVth century).

- A sentence is true if and only if it signifies *sicut est* and if it not **self-falsifying**. Self-falsifying sentences are always false.
- The Liar is self-falsifying, so it is false.
- **Consequence of Swyneshed’s definition of truth.**
  - ϕ : ϕ is false.
  - ψ : ϕ is not false.
  - ϕ is false as it is self-falsifying. But then ψ is false, too. But ϕ and ψ are contradictories.
Heytesbury.

William Heytesbury (c.1310-1372).

- 1335. *Regulae solvendi sophismata*.

- The source of the paradox according to Heytesbury: The Liar “\( \varphi : \varphi \) is false” is only paradoxical since we want to retain the usual theory of signification for it. If we give that up, there is no paradox. For example, \( \varphi \) could signify “*Socrates currit*” which is free of paradoxes.

- But \( \varphi \) cannot be evaluated according to the usual theory of signification. Therefore, anyone who utters \( \varphi \) must have some other hidden signification in mind. There is no way to analyze \( \varphi \) further before we know which one this is.
Sophismata and semantics.

Some of the problems concerning the semantics of syncategoremata are part of the theory of sophismata:

\[ Socrates \ bis \ videt \ (omnem \ hominem \ praeter \ Platonem) . \]

- **Scenario 1.** Socrates enters the room and sees everyone. He leaves. Plato leaves the room. Socrates returns and sees everyone except for Plato.
  \[ Socrates \ videt \ Platonem . \]
- **Scenario 2.** Plato is not in the room at all. Socrates enters the room twice and sees everyone in there.
  \[ Socrates \ non \ videt \ Platonem . \]
Sophismata and semantics.

Some of the problems concerning the semantics of syncategoremata are part of the theory of sophismata:

\[ Socrates \text{ bis (} \text{videt omnem hominem praeter Platonem)} \text{).} \]

- **Scenario 1.** Socrates enters the room and sees everyone. He leaves. Plato leaves the room. Socrates returns and sees everyone except for Plato.
- **Scenario 2.** Plato is not in the room at all. Socrates enters the room twice and sees everyone in there.

\[ Socrates \text{ non videt Platonem}. \]
Obligationes (1).

Obligationes. A game-like disputation, somewhat similar to logic games. The origin is unclear, as is the purpose. The name derives from the fact that one of the players is “obliged” to follow certain formal rules of discourse.

Different types of obligationes.

- positio.
- depositio.
- dubitatio.
- impositio.
- petitio.
- rei veritas / sit verum.
Obligationes (2).

- William of Shyreswood (1190-1249)
- Walter Burley (Burleigh; c.1275-1344)
- Roger Swyneshed (d.1365)
- Richard Kilvington (d.1361)
- William Ockham (c.1285-1347)
- Robert Fland (c.1350)
- Richard Lavenham (d.1399)
- Ralph Strode (d.1387)
- Peter of Candia
- Paul of Venice (c.1369-1429)
Obligationes (3).

- Roger Swyneshed, *Obligationes* (1330-1335). Radical change in one of the rules results in a distinctly different system.

**responsio antiqua**
- Walter Burley
- William of Shyreswood
- Ralph Strode
- Peter of Candia
- Paul of Venice

**responsio nova**
- Roger Swyneshed
- Robert Fland
- Richard Lavenham
**positio** according to Burley (1).

- Two players, the **opponent** and the **respondent**.
- The **opponent** starts by positing a *positum* \( \varphi^* \).
- The **respondent** can “admit” or “deny”. If he denies, the game is over.
- If he admits the *positum*, the game starts. We set \( \Phi_0 := \{ \varphi^* \} \).
- In each round \( n \), the **opponent** proposes a statement \( \varphi_n \) and the **respondent** either “concedes”, “denies” or “doubts” this statement according to certain rules. If the **respondent** concedes, then \( \Phi_{n+1} := \Phi_n \cup \{ \varphi_n \} \), if he denies, then \( \Phi_{n+1} := \Phi_n \cup \{ \neg \varphi_n \} \), and if he doubts, then \( \Phi_{n+1} := \Phi_n \).
**positio** according to Burley (2).

- We call $\varphi_n$ pertinent (relevant) if either $\Phi_n \vdash \varphi_n$ or $\Phi_n \vdash \neg \varphi_n$. In the first case, the respondent has to concede $\varphi_n$, in the second case, he has to deny $\varphi_n$.

- Otherwise, we call $\varphi_n$ impertinent (irrelevant). In that case, the respondent has to concede it if he knows it is true, to deny it if he knows it is false, and to doubt it if he doesn’t know.

- The opponent can end the game by saying *Tempus cedat.*
**Example 1.**

<table>
<thead>
<tr>
<th>Opponent</th>
<th>Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td>I posit that Cicero was the teacher of Alexander the Great: $\varphi^*$.</td>
<td>I admit it. $\Phi_0 = {\varphi^*}$.</td>
</tr>
<tr>
<td>Cicero was Roman: $\varphi_0$.</td>
<td>I concede it. Impertinent and true; $\Phi_1 = {\varphi^*, \varphi_0}$.</td>
</tr>
<tr>
<td>The teacher of Alexander the Great was Roman: $\varphi_1$.</td>
<td>I concede it. Pertinent, follows from $\Phi_1$.</td>
</tr>
</tbody>
</table>
Example 2.

<table>
<thead>
<tr>
<th>Opponent</th>
<th>Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td>I posit that Cicero was the teacher of Alexander the Great: ( \varphi^* ).</td>
<td>I admit it.</td>
</tr>
<tr>
<td>The teacher of Alexander the Great was Greek: ( \varphi_0 ).</td>
<td>( \Phi_0 = { \varphi^* } ).</td>
</tr>
<tr>
<td>Cicero was Greek: ( \varphi_1 ).</td>
<td>I concede it.</td>
</tr>
<tr>
<td></td>
<td>Impertinent and true; ( \Phi_1 = { \varphi^*, \varphi_0 } ).</td>
</tr>
<tr>
<td></td>
<td>I concede it.</td>
</tr>
<tr>
<td></td>
<td>Pertinent, follows from ( \Phi_1 ).</td>
</tr>
</tbody>
</table>
Example 3 (‘‘order matters!’’)

<table>
<thead>
<tr>
<th>Opponent</th>
<th>Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td>I posit that Cicero was the teacher of Alexander the Great: ( \varphi^* ).&lt;br&gt;The teacher of Alexander the Great was Roman: ( \varphi_0 ).</td>
<td>I admit it. ( \Phi_0 = { \varphi^* } ).&lt;br&gt;I deny it. Impertinent and false; ( \Phi_1 = { \varphi^*, \neg \varphi_0 } ).&lt;br&gt;I deny it. Pertinent, contradicts ( \Phi_1 ).</td>
</tr>
</tbody>
</table>
Properties of Burley’s *positio*.

- Provided that the *positum* is consistent, no disputation requires the *respondent* to concede $\varphi$ at step $n$ and $\neg \varphi$ at step $m$.

- Provided that the *positum* is consistent, $\Phi_i$ will always be a consistent set.

- It can be that the *respondent* has to give different answers to the same question (Example 4).

- The *opponent* can force the *respondent* to concede everything consistent (Example 5).
Example 4.

Suppose that the **respondent** is a student, and does not know whether the King of France is currently running.

<table>
<thead>
<tr>
<th>Opponent</th>
<th>Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td>I posit that you are the Pope or the King of France is currently running: $\varphi^*$</td>
<td>I admit it. $\Phi_0 = {\varphi^*}$.</td>
</tr>
<tr>
<td>The King of France is currently running: $\varphi_0$</td>
<td>I doubt it. Impertinent and unknown; $\Phi_1 = {\varphi^*}$.</td>
</tr>
<tr>
<td>You are the Pope: $\varphi_1$.</td>
<td>I deny it. Impertinent and false; $\Phi_2 = {\varphi^*, \neg \varphi_1}$.</td>
</tr>
<tr>
<td>The King of France is currently running: $\varphi_2 = \varphi_0$.</td>
<td>I concede it. Pertinent, follows from $\Phi_2$.</td>
</tr>
</tbody>
</table>
Example 5.

Suppose that $\varphi$ does not imply $\neg \psi$ and that $\varphi$ is known to be factually false.

<table>
<thead>
<tr>
<th>Opponent</th>
<th>Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>$\varphi$</td>
</tr>
<tr>
<td>$\neg \varphi \lor \psi$</td>
<td>$\neg \varphi \lor \psi$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>$\psi$</td>
</tr>
</tbody>
</table>
**positio according to Swyneshed.**

- All of the rules of the game stay as in Burley’s system, except for the definition of *pertinence*.

- In Swyneshed’s system, a proposition $\varphi_n$ is *pertinent* if it either follows from $\varphi^*$ (then the *respondent* has to concede) or its negation follows from $\varphi^*$ (then the *respondent* has to deny). Otherwise it is impertinent.
Properties of Swyneshed’s *positio*.

- Provided that the *positum* is consistent, no disputation requires the *respondent* to concede $\varphi$ at step $n$ and $\neg \varphi$ at step $m$.

- The *respondent* never has to give different answers to the same question.

- $\Phi_i$ can be an inconsistent set (Example 6).
Example 6.

Suppose that the respondent is a student in Paris, and not a bishop. Write $\psi_0$ for “You are in Rome” and $\psi_1$ for “You are a bishop”.

| Opponent | Respondent | $\Phi_0 = \{\psi_0 \lor \psi_1\}$.
|-----------|-------------|---------------------------------|
| I posit that you are in Rome or you are a bishop: $\psi_0 \lor \psi_1$ | I admit it. | Pertinent, follows from $\Phi_0$; $\Phi_1 = \{\psi_0 \lor \psi_1\}$.
| You are in Rome or you are a bishop: $\psi_0 \lor \psi_1$ | I concede it. | Impertinent, and true; $\Phi_2 = \{\psi_0 \lor \psi_1, \neg \psi_0\}$.
| You are not in Rome: $\neg \psi_0$ | I concede it. | Impertinent, and true; $\Phi_3 = \{\psi_0 \lor \psi_1, \neg \psi_0, \neg \psi_1\}$.
| You are not a bishop: $\neg \psi_1$ | I concede it. |                              |

$\Phi_2$ is an inconsistent set of sentences.
positio according to Kilvington.

Richard Kilvington (d.1361).

- *Sophismata*, c.1325.
- *obligationes* as a solution method for sophismata.

He follows Burley’s rules, but changes the handling of impertinent sentences. If $\varphi_n$ is impertinent, then the respondent has to concede if it were true if the *positum* was the case, and has to deny if it were true if the *positum* was not the case.
impositio.

- In the *impositio*, the **opponent** doesn’t posit a *positum* but instead gives a definition or redefinition.

- **Example 1.** “In this *impositio*, *asinus* will signify *homo*.

- **Example 2.** “In this *impositio*, *deus* will signify *homo* in sentences that have to be denied or doubted and *deus* in sentences that have to be conceded.

Suppose the **opponent** proposes “*deus est mortalis*”.

- If the **respondent** has to deny or doubt the sentence, then the sentence means *homo est mortalis*, but this is a true sentence, so it has to be conceded. Contradiction.

- If the **respondent** has to concede the sentence, then the sentence means *deus est mortalis*, but this is a false sentence, so it has to be denied. Contradiction.

- An *impositio* often takes the form of an insoluble.