UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2004/2005; 1st Semester
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Homework Set # 12 *Deadline:* December 8th, 2004

Exercise 37 (8 points total).

Find wellorders $\mathbf{W}$ and $\mathbf{W}^*$ such that $\mathbf{W} \oplus \mathbf{W}^*$ is not isomorphic to $\mathbf{W}^* \oplus \mathbf{W}$ and explain why (3 points). Similarly, find wellorders $\mathbf{W}$ and $\mathbf{W}^*$ such that $\mathbf{W} \otimes \mathbf{W}^*$ is not isomorphic to $\mathbf{W}^* \otimes \mathbf{W}$ and explain why (3 points). You can choose one wellorder to be finite. Why can't both wellorders be finite in such an example (2 points)?

Exercise 38 (4 points).

The ordinal ω_1^{CK} is sometimes called "the least admissible ordinal" and has an equivalent description in terms of an axiom system called "Kripke-Platek Set Theory" KP. Find out what this means and give a brief (two to four sentences) description of the connection between the ordinal and KP.

Exercise 39 (9 points total).

Let $\mathcal{L} := \{\dot{0}, \dot{1}, \dot{+}, \dot{\times}\}$ be the language of fields (*i.e.*, $\dot{0}$ and $\dot{1}$ are 0-ary function symbols, and $\dot{+}$ and $\dot{\times}$ are binary function symbols; if you don't know what a field is, please find out. On the other hand, the details are not really important for this exercise.) For a variable x, we define a term $x \cdot n$ by recursion: $x \cdot 1 := x$ and $x \cdot (n+1) := (x \cdot n)\dot{+}x$. Let χ_n be the formula $\exists x(\neg(x = \dot{0}) \wedge x \cdot n = \dot{0})$. (Note that χ_1 never holds.) Let $n \geq 2$. We say that a field $\mathbf{K}$ has **characteristic** $\leq n$ if $\mathbf{K} \models \chi_n$, that it has **characteristic** n if n is least such that it has characteristic $\leq n$, and that it has **characteristic zero** if for all n, $\mathbf{K} \models \neg\chi_n$.

(1) Prove: If φ is an $\mathcal{L}$-sentence that holds in all fields of characteristic zero, then there is some natural number n such that φ holds in all fields of characteristic n (4 points).
(2) If $\mathbf{K}_n$ is a field of characteristic n and U is a nonprincipal ultrafilter on $\mathbb{N}$, what is the characteristic of the ultraproduct $\mathrm{Ult}(\langle \mathbf{K}_n\,;\, n \in \mathbb{N}\rangle, U)$ (5 points)?

Exercise 40 (4 points).

Explain why Kripke models $\mathbf{F}$ modelling the natural language notion of "it is allowed that" (*i.e.*, $\mathbf{F} \models \Diamond\varphi$ implies "φ is allowed") are not in general reflexive.