Universiteit van Amsterdam
Institute for Logic, Language and Computation

Recursion Theory

2003/2004; 1st Semester
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Homework Set # 5. Deadline: October 30th, 2003

Exercise # 1 *(Soare, I.4.24; p. 23).*
Show that the partial recursive functions are not closed under μ, *i.e.*, there is a partial recursive function f such that $x \mapsto \mu y(f(x,y) = 0)$ is not partial recursive. (Note that rule (VI) does **not** state closure under μ, but closure under μ with a constraint.)
Hint.

$$f(x,y) = \begin{cases} 1 & \text{if } y = 0 \text{ and } \varphi_x(x)\uparrow, \\ 0 & \text{otherwise.} \end{cases}$$

Exercise # 2.
Prove that

(1) there is a natural number n such that $W_n = \{x\,;\, n|x\} = \{x\,;\, n \text{ divides } x\}$,
(2) there is a natural number n such that φ_n is the constant function const_n,
(3) there is a natural number n such that φ_n is the polynomial function $f(x) = x^n$.

Exercise # 3 *(Soare, I.4.27; p.23).*
Prove that Fin $\leq_1$ Cof.
Hint. Use

$$f(x,y) := \begin{cases} \uparrow & \text{if } W_{x,y+1} \backslash W_{x,y} \neq \varnothing, \\ 0 & \text{otherwise.} \end{cases}$$